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A Unit Root Model for Trending Time-series Energy Variables

P.K. Narayan and R. Liu

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A Unit Root Model for Trending Time-Series Energy Variables

Paresh Kumar Narayan^{[1](#page-1-0)} and Ruipeng Liu

ABSTRACT

In this paper, we propose a GARCH-based unit root test that is flexible enough to account for; (a) trending variables, (b) two endogenous structural breaks, and (c) heteroskedastic data series. Our proposed model is applied to a range of time-series, trending, and heteroskedastic energy variables. Our two main findings are: first, the proposed trend-based GARCH unit root model outperforms a GARCH model without trend; and, second, allowing for a time trend and two endogenous structural breaks are important in practice, for doing so allows us to reject the unit root null hypothesis.

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¹ Alfred Deakin Professor, Centre for Economics and Financial Econometrics Research, Faculty of Business and Law, Deakin University, Melbourne, Australia. Email: narayan@deakin.edu.au.

I. Introduction

Testing for the null hypothesis of a unit root in energy variables has become a popular strand of research in energy economics. Beginning with Narayan and Smyth (2007), and as a result motivated by the need for understanding the unit root nature of energy variables, the energy unit root literature has surged. A survey of this literature has been undertaken by Smyth (2013) and more recently by Smyth and Narayan (2014). From this survey, what becomes clear is that the energy unit root literature has progressed from using simple univariate unit root tests to using structural break unit root test to panel data unit root tests both with, and without, structural breaks. The success of this literature rests heavily on the econometric tests developed. This is important to recognise because the current paper belongs to this category of studies.

In this paper, we propose a new unit root model for testing the non-stationarity of energy variables. We propose a GARCH-based two endogenous structural break unit root test. Our main contribution is that we address important statistical issues that matter for unit root testing in energy variables which the literature has so far ignored. To be more specific, there are three statistical issues with which we deal. In equal order of importance, the first one is a GARCH model with a time trend. Here, we extend the GARCH unit root model for a unit root developed by Narayan and Liu (NL, 2011; 2013). The NL model does not, however, include a time trend, which can be costly in practice because a time trend if present can be a source of power to reject the unit root null hypothesis.

Almost all energy variables seem to be characterised by a time-trend yet so far no attempt has been made to account for trending energy variables in testing for a unit root. This is surprising because most energy variables we deal with, including the oil price data widely used for forecasting (see Narayan et al. 2014), have a clear upward trend. Indeed the observation that a linear deterministic trend should be accommodated in time series data and its practical relevance is nothing new; in fact, such an observation was made in a seminal paper by Phillips and Perron (1988). Diebold and Kilian (2000) argue that since most time series are characterised by trending behaviour, failure to model the series properly (that is through including a time trend in a unit root model) will lead to bias estimation. Specifically to energy variables that we use; it is clear that energy variables are trending as can be seen from Figures I, II, and III. On the whole energy variables tend to grow over time. Several studies in energy economics show that growth in energy variables over time is a result of income growth, population growth, and urbanisation; see references cited in a recent survey paper by Smyth and Narayan (2014). Moreover, in a recent study, Westerlund, Norkute, and Narayan (2014) demonstrate the econometric relevance of accounting for a time-trend for a panel of 17 commodities, which includes crude oil and natural gas. In their theoretical and Monte Carlo analysis, time-trend is seen as important in testing for what they term the futures efficient market hypothesis.

The practical importance of time trends in energy variables have also been documented by several studies. BP (2013), for instance, argue that time trends matter for energy variables. In particular, the report argues that oil prices follow a long-run trend and these long-run trend movements dictate demand and supply responses in the oil market. In terms of profitability, purely from an investor trading point of view, Narayan, Narayan and Sharma (2013) show that a time-trend analysis results in investors making amongst the most profits from investing in the oil market. Moreover, Narayan, Ahmed, and Narayan (2014) use a trend-based momentum trading strategy and show that a portfolio of 19 commodities, which includes oil, offer statistically significant portfolio returns to investors over time (1983 to 2012) and these results hold regardless of whether one uses daily or monthly time-series data. The key implication of ignoring a time-trend in energy variables, therefore, points towards a model misspecification which comes with not one but multiple costs. These are: (i) allowing energy prices to fluctuate but not around a broken trend as shown by several studies (Smyth, 2013; Narayan, Narayan, and Popp, 2010; Agnolucci and Venn, 2011; and Apergis and Payne, 2010); and (ii) loss of information content, whether from a statistical point of view (see Diebold and Kilian, 2000) or from an economic significance point of view (see Narayan, Narayan, and Sharma, 2013 and Narayan, Ahmed and Narayan, 2014), which results from ignoring the underlying movements in energy prices that are ignored when the time-trend is not included in the model.

Our treatment of a time-trend in the model also implies that we take into account as much information as available for those energy variables; in particular, the preference for a time-trend gives us a rich characterisation of the energy data through impounding in our model the underlying value of those variables.^{[2](#page-4-0)} Because, by virtue of a time-trend, we have as much information on energy variables as statistically possible, it makes more sense to test for their integration property.^{[3](#page-4-1)}

What this means is that if we were to ignore a time-trend in the unit root model, the result, as explained by Diebold and Kilian (2000), will be a misspecified model. Our proposed unit root model, therefore, is free from such misspecified model criticism and, as a

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² The relevance of information content in the time-trend can be demonstrated by using a trend-cycle decomposition and then testing for the unconditional correlation between the original price series and its trend component. We do this. Specifically, using the Hodrick-Prescott (HP, 1997) technique we decompose each energy price variable at each of the three data frequencies into its trend and cycle components and then test for unconditional correlation between energy price and its trend. We also test the null hypothesis that this unconditional correlation is zero. Several statistics are important here. First, we notice that all correlations are close to one; with monthly data, the range is 0.76 to 0.86, with weekly data the range is 0.96 to 0.99, while with daily data the unconditional correlations across the seven series are in excess of 0.99. Second, the correlations increase with data frequency, demonstrating higher information content (possibly) with high frequency data. Third, all correlations are statistically different from zero. Detailed results are available from the corresponding author upon request. The main implication of this exercise is that the trend component in energy prices is important and should not be ignored. We feel safe by not ignoring the trend component. ³ One referee of this journal highlighted other characteristics of time-series data that may possibly be modelled apart from time-trend. These could be cyclical components or seasonal components. These are interesting avenues for future research. In fact cyclical components have been shown in energy variables by Narayan, Narayan, and Smyth (2011); alas, not much work has been done on the cyclical components of energy variables. Again, this is an area for future research. Indeed, seasonal components in time-series data have been explored from an econometric point of view and there is scope for active research on this front from the energy economics point of view. For a recent development of a seasonal unit root test with trending

data that can be applicable to the energy variables, since we show that these variables are trending, see Costantini, Narayan, Popp, and Westerlund (2015). At this point it is also important to recognise that the subject of our paper is time trend structural break unit root tests relating to the strand of the literature where the focus has been on structural break time trend models.

result, is also consistent with recent structural break time series unit root models; for a survey of these models, see Narayan and Popp (2013).

Our second contribution relates to modelling high frequency data and the resulting heteroskedasticity in the data.^{[4](#page-5-0)} Recent empirical analysis of unit roots in energy variables has started to entertain applications involving high frequency data, such as monthly, weekly and daily data.^{[5](#page-5-1)} While the use of high frequency data is welcomed in the sense that they allow investigators to model more information content, the cost is that high frequency data introduces an additional statistical issue, namely, heteroskedasticity. In light of this, we ensure that in proposing a trend-based unit root model, we keep the model within the family of GARCH models, which were developed in the financial econometrics literature to account for heteroskedasticity in financial time-series data.

Our contention is that by allowing for these features of the data we have on hand a relatively more powerful unit root model, allowing us to more accurately model the unit root behaviour of energy variables. We show this to be the case through using Monte Carlo simulations. From this exercise, we find, for example, that our proposed trend-GARCH unit root model is not only correctly sized but is relatively more powerful and detects structural breaks more accurately compared to ADF tests and GARCH unit root tests without structural breaks and without a time trend.

We undertake an application of the trend-GARCH model using seven energy (spot) price series; namely, crude oil, gasoline, heating oil, diesel, jet fuel, propane and natural gas. To provide robustness of our results we consider two data frequencies—monthly and weekly. These data frequencies appear to be most popular in applications. Our findings suggest strong

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 4 In fact, the recent literature has begun using high frequency data and have as a result show heteroskedasticity to be an issue in unit root testing. Therefore, these studies (see Mishra and Smyth, 2014a, b and Salisu and Mobolaji, 2013) have used the Narayan and Liu (2011, 2013) GARCH unit root test with two structural breaks.

⁵ For an application of a unit root test (without structural breaks) to daily oil price, see Westerlund and Narayan (2013); and, for a search of multiple structural breaks in daily oil price data, see Narayan et al. (2013)

evidence of stationarity for prices of five out of seven spot prices series; the exception being propane for which we could not reject the unit root null hypothesis regardless of the data frequency. For the last spot price—crude oil—we find we could only reject the unit root null hypothesis when using weekly data and not at monthly data frequency, suggesting that at least for crude oil the results are data frequency dependent.

We organise the balance of the paper as follows. The next section explains the methodology. Section III presents the data, the simulation results and results from the trend-GARCH model. The final section provides some concluding remarks.

II. Empirical Framework

In this section, we present our empirical framework, which extends the modelling framework in Narayan and Liu (2011), who propose a GARCH-based unit root model that accounts for two endogenous structural breaks. We extend this model by introducing into it a time trend. Several studies (see Narayan and Smyth, 2007; Narayan et al., 2008) have shown that the mean, variance, and co-variance of energy series tends to change over time. This implies that there is a tendency for energy variables to attain a new equilibrium when exposed to shocks, both political and economic shocks that influence energy prices, its consumption and production. In light of this finding, it is important to start by assuming that the energy variable on hand is a non-stationary process. We propose a trend-GARCH (1, 1) unit root model with two endogenous structural breaks, which has the following form:

$$
y_t = a_0 + a_1 t + \rho y_{t-1} + \sum_{i=1}^k D_i B_{it} + \varepsilon_t \qquad (1)
$$

Here, $B_{it} = 1$ for $t \ge T_{Bi}$ otherwise $B_{it} = 0$; T_{Bi} are the structural break points, where $i = 1, 2, 3...k$; D_i is break dummy coefficient. At this point it is imperative to deal with our choice of two structural breaks. As a guide to this choice, we draw on the Bai and Perron (BP, 2003) multiple structural break test. The key advantage of the BP test is that it allows us to search for a maximum of five structural breaks in a time series of data. Applying the BP test reveals that the most common number of maximum breaks are two; of the 21 time-series of energy variables on hand (across the three data frequencies), for 14 series there are exactly two structural breaks while for the remaining seven series there are at least two breaks with no more than three breaks (see Table IV). Taking these structural breaks as a guide, we allow for either two or three structural breaks in the energy price series depending obviously on data frequency and the actual price series.

Moreover, ε_t follows the first-order generalized autoregressive conditional heteroskedasticity model, denoted as GARCH $(1, 1)$ and has the following representation:

$$
\varepsilon_t = \eta_t \sqrt{h_t} \;, \; h_t = \varkappa + \alpha \varepsilon_{t-1}^2 + \beta h_{t-1} \tag{2}
$$

Here $\alpha > 0$, $\alpha \ge 0$, $\beta \ge 0$, and η_t is a sequence of independently and identically distributed random variables with zero mean and unit variance.

Since the break time is unknown, T_{Bi} ($i = 1, 2, 3...$ k) in Equation (1) has to be replaced by their estimates \hat{T}_{Bi} . We conduct the estimation by using a sequential procedure, that is, we search for the first break date according to the maximum absolute t-value of the break dummy coefficient D_1 , hence we obtain:

$$
\hat{T}_{B1} = \arg \max_{\hat{T}_{B1}} \left| t_{\hat{D}_1} \left(T_{B1} \right) \right| \tag{3}
$$

Imposing the first break estimate \hat{T}_{B1} , we estimate the second break date \hat{T}_{B2} such that

$$
\hat{T}_{B2} = \arg \max_{\hat{T}_{B2}} \left| t_{\hat{D}_2} \left(\hat{T}_{B1}, T_{B2} \right) \right| \tag{4}
$$

The way we choose the two endogenous structural breaks, as depicted in Equations (3) and (4) is consistent with the approach proposed by Narayan and Popp (2010). This approach to break date selection has also been shown to work impressively in Monte Carlo simulations by Narayan and Popp (2013).

III. Empirical Findings

Our findings are divided into three parts. In the first part, we provide a description of the data series we model. In the second part, we present and discuss our simulation results on the importance and relevance of our proposed two endogenous break trend-GARCH unit root test.

In the final part of this section, we undertake an application of the unit root test using time-series monthly, weekly, and daily data. There are two reasons we use three different frequencies of data. Our first reason is about confirming the robustness of our results. In hypothesis testing, it is imperative to get a good impression of how robust the results are. In this regard, our motivation is a recent paper on the role of data frequency; Narayan et al. (2014) show that results regarding profitability of the commodity market (including energy commodities) tends to be data frequency dependent. Our second reason is related to the new test. Since there is a new test and naturally it has not been applied to any high frequency data, it is imperative to subject it to different data frequencies. In this regard, our choice of monthly, weekly, and daily are based on their popularity in the applied energy literature.

A. Data

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Our focus is to understand the seven time-series data on energy variables that we model in this paper. Our sample of variables include seven energy variables; namely, Cushing OK Crude Oil Spot FOB (01/1986 to 03/2014), New York Harbor Conventional Gasoline Regular Spot Price FOB (0[6](#page-8-0)/1986 to 03/2014)⁶, New York Harbor No. 2 Heating Oil Spot Price FOB (06/1986 to 03/2014), Los Angeles, CA Ultra-Low Sulfur CARB Diesel Spot Price (04/1996 to 03/2014), U.S. Gulf Coast Kerosene-Type Jet Fuel Spot Price FOB (04/1990 to 03/2014), Mont Belvieu, TX Propane Spot Price FOB (07/1992 to 03/2014), and

⁶ We did not choose the Los Angeles Reformulated RBOB Regular Gasoline Spot Price as this data is only available back to June 2003.

Natural Gas Spot Price (01/1997 to 03/2014). All prices are quoted as Dollars per Barrel except with Gas price quoted as Dollars per Million Btu. All data are obtained from the U.S Energy Information Administration online [\(http://www.eia.gov\)](http://www.eia.gov/). The start and end dates vary by price series and by data frequency. To make this clear, in Table I we present for each monthly (Panel A), weekly (Panel B), and daily (Panel C) price data series the start and end dates and conclude this table by reporting in the last column the number of observations for each price series. Reading the last column then, we observe that for monthly data series the number of observations fall in the [206, 338] range while for weekly data we have more observations and therefore the range is 897 to 1473 observations.

Next we plot the data. The seven price series appear in Figures I, II and III for monthly weekly, and daily data, respectively. There are two rather obvious features of the data gained just through a visual inspection, regardless of data frequency. First, we notice that these energy prices are trending; therefore, it seems clear that prices are characterised by a time trend. Second, there are obviously signs that the data series have undergone structural changes. This implies that structural breaks are a feature of the energy price data set and therefore accounting for these structural breaks in testing for a unit root will be ideal and most likely will be a source of power gain in rejecting the unit root null hypothesis.

To further understand our data, we report in Table II, selected descriptive statistics. The first thing to note is that average prices across the three data frequencies are fairly similar as expected. All price series, regardless of data frequency, seem to have positive skewness. The ADF test, where serial correlation is controlled through lags of the dependent variable obtained by using the Schwarz Information criterion, suggests that the unit root null hypothesis cannot be rejected at the 5% level of significance give the critical value of 3.42. Therefore, regardless of the data frequency energy price data series are non-stationary.

We turn to heteroskedasticity now. To test for heteroskedasticity, we run an autoregressive (AR) model of weekly and monthly energy price series with 12 lags, namely an $AR(12)$ model. We filter these price series through taking the residuals from this $AR(12)$ model and testing the null hypothesis that there is 'no ARCH' effects. Given the 5% critical value of 19.68, we cannot reject the null hypothesis only for the propane price at the monthly frequency. Although at the weekly frequency, we can comfortably reject the null for propane. Therefore, it is clear that our energy price data is characterised by heteroskedasticity.

As a final point, we attempt to confirm the statistical importance of a time trend. To do this, we run OLS regressions of price variables on a time trend. The coefficients on the time trend and their p-values associated with the null hypothesis that the trend is zero are reported in the last three rows of each of the three panels in Table II. Specifically, we run three different types of regression models. The first model, denoted 'Trend' is a simple OLS model which regresses energy price over time on a constant and a time trend. The second model is the "Trend" model augmented with structural breaks obtained from our GARCH structural break test; we call this "Trend1". The third model ("Trend2") is actually an augmented version of "Trend1" in that it includes the squared of the price return variable. Two observations are key here. First, the time trend coefficient turns out to be positive. This sign effect suggests that energy variables have grown over time. Second, it turns out that the null hypothesis is mostly rejected at the 1% level, suggesting that regardless of the data frequency used and regardless of the time-trend model used, a time trend is an important feature of the energy price data. This statistical evidence is consistent with what we observed from simple time-series plots of the energy price data in Figures I-III.

B. Simulation Results

We start with the critical value. The critical values are derived for different break dates based on 50,000 replications. The critical values are reported in Table III. These critical values are generated at the 5% level of significance for different GARCH orders, 'a' and 'b'. Three different GARCH orders are considered: namely, (0.05, 0.95), (0.45, 0.5), and (0.9, 0.05),

denoting a highly persistent, medium persistent, and weakly persistent variance. In addition, six different break date combinations are considered; for example a combination of (0.2, 0.4) denotes that two break dates occur early in the sample at the $20th$ and $40th$ percentile. Similarly, a break combination of (0.6, 0.8) suggests two break dates occurring at the second half of the sample. Three specific sample sizes are considered; that is, we set $T = 150, 250$, and 500.

Our main observations from the critical values are twofold and can be summarised as follows. First, as expected the critical value declines (in absolute terms) with sample size. Second, critical values (in absolute terms) decline as the order of variance persistence declines, suggesting critical values are variance persistent dependent. When GARCH orders are (0.05, 0.95) the most usual case with high frequency data, critical values are highest when break dates occur early in the sample at 20% and 40% of the sample in the case when T=150; however, critical values (in absolute terms) are the lowest when $T = 500$. This seems to imply that the critical values are dependent on the break dates, which subsequently impacts on the decision regarding the null hypothesis of a unit root.

We now examine the empirical size and power of our test. The results are reported in Figure IV, which is based on 5000 replications and contains a sample size of 500 observations. Due to the space limitations, we only present the case for combination of GARCH parameter values being [0.05, 0.95] and break dates occurring at (0.2, 0.6). The results for other combinations of GARCH parameters and break dates are qualitatively similar and are available upon request. We analyse the test properties for our proposed model and when: (i) the break dates are known (exogenous cases); (ii) GARCH model with breaks but no time trend proposed by Narayan and Liu; (iii) the conventional ADF test; and (iv) the GARCH unit root test without breaks and time trend proposed by Cook (2008).

The main results are as follows. We begin with size properties of these competing tests. These are plotted in the upper left panel. The first thing we observe is that the ADF test and the GARCH test of Cook (2008) without breaks and a time trend are undersized, which becomes severe as the size of break increases. When we compare this outcome with our proposed GARCH structural break with time trend model, regardless of whether the break dates are chosen exogenously or endogenously, the size properties are close to the nominal 5% level. Therefore, as long as a time trend is included, the manner in which structural breaks are chosen does not make the test unstable; they remain correctly sized.

Next we evaluate the power of our test under the null hypothesis H₀: $\rho = 0.9$. The performance outcome is plotted in the lower left panel. Consistent with size properties, we find that our proposed test, regardless of whether the structural breaks are exogenously or endogenously determined, outperforms the competition. The ADF test and the GARCH unit root test without a time trend and without structural breaks have the lowest power and the power converges to zero when break size increases.

In the second column of Figure III, we report the frequency of detecting the break dates. Here, we run a horse race between our proposed test and the Narayan and Liu (2011, 2013) tests because these are the only two GARCH-based unit root tests that depend on structural breaks. We observe that the probability of detecting the actual break dates significantly increases with size of break date, and it is close to 100% for medium and largesized breaks for our proposed test, suggesting that the model detects breaks accurately for both null H₀: $\rho = 1$, and H₀: $\rho = 0.9$. We also find the performance of our proposed test is superior compared with the test without trend for small and medium-sized breaks. On the whole, from the exercise it is clear that allowing for a time trend improves the ability to detect structural breaks accurately.

C. Test of the Unit Root Null Hypothesis

The trend-GARCH unit root test that accounts for two structural breaks is implemented using the seven energy price series. Before we read these results, it is important to recap that the

ADF test on each of the seven price series, using monthly, weekly, and daily data, revealed energy prices to be non-stationary. The trend-GARCH results are presented in Table V. Panel A contains results based on monthly data, panel B has results for weekly data and panel C contains results based on daily data. Specifically, we report results on each of the break dates, which range from two to three; the t-test statistic used to examine the null hypothesis of a unit root, its coefficient beta; and the coefficient on alpha—that is, the constant term in the model. In the last row to get a feel of how long shocks impact the price series, we also report the half-life (computed as $\ln(0.5)/\ln(\alpha+\beta)$), which essentially tells us how many months, weeks, and days it takes for the impact of the shock to be halved.

We begin with monthly data. The unit root null hypothesis is rejected for five out of seven energy price series. The series that clearly appear stationary (at the 1% level) are gasoline spot price, heating oil spot price, diesel spot price, jet fuel spot price, and natural gas spot price, and crude oil at 10% level. Based on monthly data propane spot price is the only series for which the unit root null hypothesis cannot be rejected. When we consider weekly data, we observe that five of the seven series are stationary. However, with weekly data the two non-stationary series are heating oil and propane. With daily data by comparison, we find that only two of the seven series are stationary. Therefore, what we notice is that the rejection rate of the unit root null hypothesis declines with data frequency. Clearly, therefore, data frequency does matter for unit root testing.[7](#page-13-0)

We also undertake some additional results of the unit root test using other popular methods. Specifically, for this exercise we consider four additional tests; (i) the GARCH unit

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⁷ The estimates of various structural breaks seem to align with global events that impact the energy market. For an instance, the first structural break found for crude oil price is September 1990, which is mainly due to the "Gulf Crisis" which resulted when Iraq invaded Kuwait. The break dates in 2004 and 2005 relate to OPEC's agreement to reduce its official production by 1 million barrels-a-day beginning in April 2004. The break dates in 2003 relates to the Iraqi invasion and the merger of Russian oil company TNK with BP and splitting ownership of the newly formed TNK-BP 50/50. The second break date in 2008 relates to recession in the United States. The third break date in 2011 relates to the Japanese tsunami and the meltdown at Fukushima Daiichi nuclear power plant, which created a regional disaster.

root test of Cook (2008), which doesn't include a time trend and structural breaks; (ii) the GARCH two-structural break test of Narayan and Liu (2013), which does not include a time trend; and the three versions of the Narayan and Popp (2010) two structural break test, which neither includes a time trend nor heteroskedasticity. These results are reported in Table VII. Generally, as expected, we notice that across all three data frequencies, the results from these alternative tests are weak and the cause of this weakness seems to be a function of the fact that these studies either do not allow for a time trend (as in the case of the existing GARCH unit root models), they do not allow for structural breaks (as is the case with model proposed by Cook (2008), or they do not model heteroskedasticity as in the case of the Narayan and Popp (2010) test. Indeed what seems also clear is that the Narayan and Popp (2010) test that allows for both time trend and structural breaks (although it ignores heteroskedasticity) performs closest to our trend-GARCH structural break test, suggesting the relative importance of time trends in time series data in general.

D. Robustness test

In order to check if other GARCH specifications lead to qualitatively similar results, we undertook some robustness tests to give credence to our findings and conclusions. Specifically, we tested the robustness of our results by replacing GARCH (1, 1) with GARCH (1, 2) and GARCH (2, 2) specifications. The results are reported in Table VIII. While we do observe slight changes in the results between GARCH (1, 2) and GARCH (2, 2) specifications, they seem to matter more when using monthly data than weekly and daily data.

V. Concluding Remarks

Motivated by a rich volume of studies that test for the unit root properties of energy variables in this paper we propose a new trend-GARCH unit root test that accounts for two endogenous structural breaks. The main advantage of our proposed test is that, while in keeping with the family of GARCH unit root test, such as the two break test developed by Narayan and Liu that easily is able to account for any heteroskedastic data series, it is able to take issue with time-series that contain a time trend. In this regard, energy price variables are prime candidates for a time trend, as we show in this paper through taking monthly and weekly time series price data on seven energy price series.

Our proposed trend-GARCH structural break test is shown to be correctly sized amongst the competition, enjoys more power and helps one to search for structural break dates more accurately. These features are appealing as they present researchers with a tool for ensuring that the unit root null hypothesis is tested with greater precision. For this reason, the proposed test will be appealing and widely used we believe, not only in the energy economics literature but in other literatures where unit root tests demand accounting for a time trend, such as when using relatively high frequency data.

Our empirical analysis, based on monthly, weekly, and daily data, suggests that at best we can comfortably reject the unit root null hypothesis for five out of seven energy price series. Propane spot price clearly appears to be non-stationary regardless of data frequency used while there is some uncertainty with regard to the unit root property of the crude oil spot price although when we apply the trend-GARCH test to weekly oil price data the unit root null hypothesis is comfortably rejected. The least evidence of stationary is found when using daily data. On the whole, we are comfortable in recommending this test when one has on hand a relatively high frequency of data and when this data is heteroskedastic and contains a time trend. The test will be useful for policy makers in understanding whether or not shocks to energy prices are permanent or transitory. And, as we show using half-life analysis, one can also deduce the length of time it takes for the shock to diminish. These are important policy considerations.

In closing, it is worth highlighting that our paper does not claim to address all issues in unit toot testing. Indeed future research can do more by extending our work. For example, one avenue for future research could be extending our model to different classes of GARCH models. Although this will be computational demanding, it remains an area for future research.

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Figure I: A plot of monthly energy price data

This figure plots monthly data over the period 1986 to 2014 seven energy (spot) price series; namely, crude oil, gasoline, heating oil, diesel, jet fuel, propane and natural gas.

Figure II: A plot of weekly energy price data

This figure plots weekly data over the period 1986 to 2014 seven energy (spot) price series; namely, crude oil, gasoline, heating oil, diesel, jet fuel, propane and natural gas.

Figure III: A plot of Daily energy price data

This figure plots weekly data over the period 1986 to 2014 seven energy (spot) price series; namely, crude oil, gasoline, heating oil, diesel, jet fuel, propane and natural gas.

Figure IV: A plot of size and power

This figure plots a number of important results from a Monte Carlo simulation Simulations are based on GARCH parameter values being [0.05, 0.9] and break dates occurring at $T_B = (0.2, 0.6)$. Upper panels are under H₀: $\rho = 1$, and Lower panels are under H₀: $\rho = 0.9$, respectively. The first plot is about empirical size, below which is a power plot, while break date estimation accuracy is plotted in the two right-hand side plots.

Table I: Information of data on sample size

This table reports information regarding our data sample and the number of observations. For each of the seven energy price series, the start and end dates are reported in column 2 and the number of observations for each price series is reported in the final column. Panel A reports on monthly data while panel B contains the corresponding information for weekly data. All data are obtained from the U.S Energy Information Administration online (http://www.eia.gov).

Table II: Selected descriptive statistics of the data

This table reports a range of commonly used descriptive statistics on seven energy price series; namely, crude oil, gasoline, heating oil, diesel, jet fuel, propane, and natural gas. The specific statistics reported are the mean price, its standard deviation, minimum (min.) and maximum (max.) prices, skewness and kurtosis of price, an Augmented Dicey and Fuller (ADF, 1981) unit root test that includes a time trend. The optimal lag length in the ADF regression is chosen using the Schwarz Information criterion (SIC); we begin with a maximum of eight lags and then use the SIC to select the optimal lag length. The optimal lag lengths are reported in square brackets beside the ADF t-test statistic. The last row contains a simple test for heteroskedasticity. We filter the price series using an autoregressive model with an order of 12 and then implement the Lagrange Multiplier test examining the null hypothesis of "no ARCH". The LM test statistic and the p-value (in parenthesis) are reported. The last three rows report trend coefficients and the p-value (in parenthesis); 'Trend' is an OLS regression model with time trend only; 'Trend1'is an OLS regression model with structure breaks and time trend; 'Trend2'is an OLS regression model with structure breaks and time trend plus the squared of the price return variable.

Table III: Critical values

This table reports the critical values of the GARCH two structural break unit root tests with a time trend. The critical values are generated at the 5% level for different GARCH orders, 'a' and 'b'. Three different GARCH orders are considered: namely, (0.05, 0.95), (0.45, 0.5), and (0.9, 0.05) denoting a highly persistent, medium persistent, and weakly persistent variance. In addition, six different break date combinations are considered; for example a combination of $(0.2, 0.4)$ denotes that two break dates occur early in the sample at the $20th$ and $40th$ percentile that is, while a break combination of (0.6, 0.8) suggests two break dates occurring at the second half of the sample. Three specific sample sizes are considered; that is, we set $T = 150, 250,$ and 500.

Table IV: Bai and Perron test results

The tests for the number of structure breaks are based on Bai and Perron (2003), i.e., supF tests against a sequential number of breaks using global optimizers, the critical values of sup $F(i+1|i)$ at the 10 % level are (for $i=1$ to 5.00) are: 7.04 8.51 9.41 10.04 10.58, respectively; The critical values of supF($i+1$ |i) at the 5 % level are (for i=1 to 5.00) are: 8.58, 10.13, 11.14, 11.83, 12.25, respectively; the critical values of supF(i+1|i) at the 1 % level are (for i=1 to 5.00) are: 12.29 13.89 14.80 15.28 15.76, respectively.

Panel B: Results based on weekly data

Panel C: Results based on Daily data

Table V: Unit root test results

This table presents the trend-GARCH two structural break unit root test results. Panel A contains results based on monthly data, Panel B contains results based on weekly data, and Panel C has results based on daily data. The results are generated for each of the seven energy price series; namely, crude oil, gasoline, heating oil, diesel, jet fuel, propane and natural gas. The first two (three) rows of results in each panel contain the first and second break dates followed by the trend-GARCH t-test statistic examining the null hypothesis of a unit root. The last three rows are GARCH parameters α , β and the half-lives computed. *, **, *** stand for the significances at 10%, 5%, and 1% levels. The critical values for each cases are reported in the table VI.

Panel B: Results based on weekly data

Panel C: Results based on Daily data

Table VI: Unit root test critical values (CVs)

This table reports the critical values that are used in Table V. These critical values of the GARCH two structural break unit root test with a time trend are based on the empirical data and estimates in Table V. In particular, we generate critical values specific to the sample period of data, break date magnitude, and the GARCH parameters.

Table VII: Unit root test results based on other unit root type tests

This table presents additional results based on other methods for testing the unit root null hypothesis.G1 refers to the unit root with GARCH test proposed by Cook (2008), which does not include a time trend and structural breaks; G2 refers to the GARCH two-structural break test without a time trend proposed by Narayan and Liu (2013) results; and models M0, M1, and M2 are different versions of the structural break unit root test proposed by Narayan and Popp (2010), which neither include a time trend nor model data heteroskedasticity. Results based on monthly, week and daily data are reported in Panels A, B, and C, respectively.

Table VIII: Unit root test results – GARCH (1, 2) and GARCH (2, 2)

This table presents the trend with two structural break unit root test by specifying GARCH(1, 2) and GARCH (2, 2) . Panel A contains results based on monthly data, Panel B contains results based on weekly data, and Panel C contains results based on daily data. The results are generated for each of the seven energy price series; namely, crude oil, gasoline, heating oil, diesel, jet fuel, propane and natural gas. The first row of results in each panel report the trend-GARCH (1, 2) test statistic values; The second row of results in each panel report the trend-GARCH (2, 2) test statistic values. *, **, *** stand for the significances at 10%, 5%, and 1% levels.

Panel A: Results based on monthly data							
	crude oil Spot Price	Gasoline Spot Price	Heating Oil Spot Price	Diesel Spot Price	Jet Fuel Spot Price	Propane Spot Price	Natural Gas Spot Price
GARCH(1,2)	-3.19	$-5.63***$	-3.31	$-551***$	$-4.28***$	-1.31	$-4.37***$
GARCH(2,2)	$-3.58**$	$-6.71***$	$-4.42***$	$-4.52***$	$-5.19***$	-1.56	$-4.29***$

Panel B: Results based on weekly data

Panel C: Results based on Daily data

