Faculty of Business and Law

SCHOOL OF ACCOUNTING, ECONOMICS AND FINANCE

School Working Paper - Economic Series 2006

SWP 2006/22

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August 2006

Abstract

In a principal-agent environment with moral hazard and symmetric information, having or acquiring a more informative technology lowers the cost to implement a given action. Contracting may occur after or before the principal learns her technology. We show that when the principal has or will acquire private information about her technology, (i) with ex post contracting, the value of information for the principal may be negative; and (ii) although the agent prefers that the principal has private information with ex post contracting, ex ante contracting is superior to ex post contracting by the Potential Pareto Criterion. KEYWORDS: *Moral Hazard, Principal-Agent, Informed Principal, Information, Technology.* JEL Classification: D82, D83

*I would like to thank Ed Schlee, Alejandro Manelli, and Hector Chade for their helpful comments and suggestions. The paper also benefited from comments by seminar participants at Deakin University, LaTrobe University, and the Australian Economic Society Meetings 2004.

1 Introduction

There are many relationships that can be analyzed with the principal-agent framework with moral hazard; e.g., landlord-sharecropper, insurer-insuree, owner-manager, and contractor-subcontractor. In order to induce effort, the optimal contract must place some risk on the agent's compensation. The principal's objective is to maximize profit by implementing an effort level through the choice of a compensation scheme (Holmström [7], Grossman and Hart [6]).

Several extensions have explored the effects of the agent not only choosing an unobservable action, but also of the agent having or acquiring private information (Holmström [7], Myerson [14], Sobel [19]). It is not difficult to find examples however in which the principal has private information. For example, a landlord may know the distribution of crop yields conditional on effort better than a transient sharecropper; and, an owner knows the economic and financial situations of the firm better than the manager being hired.

Following the terminology introduced by Maskin and Tirole ([12], [13]) (see also Myerson [15]), we examine a situation in which the principal may have private information about the technology; i.e., the Markov matrix relating the agent's action to observable outcomes. The probabilities of different outcomes and the returns to effort affect the contract and thus the action that the principal implements. Grossman and Hart [6] showed that if the technology is common knowledge, then the contract is second-best,[1] and the more informative the technology is, the greater is the principal's profit. Chade and Silvers [1] showed that, if the principal has private information about the technology, the latter result need not hold; i.e., there exist equilibria in which the principal with the more informative technology earns less profit than the principal with the less informative technology.

Thus, when designing a contract, each party's prior beliefs about the principal's technology are crucial in determining the equilibrium contracts and consequent payoffs. Since the principal can be worse off by having a more informative technology if this is her private information, a natural extension is to determine the value of information relating to how well the principal knows her technology.

In this paper, we characterize situations in which the principal prefers not to be able to better discern her technology when contracting occurs after she receives the signal; whereas, when contracting occurs before she receives the signal, then she prefers to be able to better discern her technology. Finally, we show that the principal, agent, and agency can gain by contracting before rather than after the principal observers the signal, by the Potential Pareto Criterion.

Our paper is related to the growing literature on principal-agent models with moral hazard and a privately informed principal (Maskin and Tirole [13], Inderst [8], Chade and Silvers [1]). In these papers, it is shown that the presence of private information for the principal makes the high type

[1] When the agent's action is observable, the first-best contract, where risk-sharing is perfect and the efficient action is implemented, is attainable. When the agent's action is unobservable, the optimal contract trades off risk-sharing benefits for greater incentives, yielding a second-best contract.

worse off. Our work is also related to the literatures that examine the value of a more informative technology (Grossman and Hart [6], Kim [10], Jewitt [9]) and the value of information (Gjesdal [5], Kim [11]). Our results imply that, when the principal has private information, the value of information for the principal with the more informative technology can be negative.

To see the implications of our results, consider, for example, insurance markets and government procurement. In such situations, because the insurer or government has considerable experience, the principal and the agent do not meet until after the principal has observed the signal of the technology. We show that the principal would we worse off if she knew more accurately her technology.

It may be possible for the agent to pay to learn what the principal knows prior to contracting. For example, an executive to whom a firm has offered a position may find it beneficial to research the firm's prospects and profitability conditional upon his effort, prior to accepting or rejecting the contract offer. Our results indicate that the principal would gain by disclosing the returns to the agent's effort, by making the technology common knowledge.

Alternatively, the principal and agent may both be ignorant of the technology, as when a franchiser expands into a new market. The franchiser could offer an ex ante contract to the franchisee, or collect data to better learn the technology and then offer an ex post contract to him. Our results imply that the franchiser would gain by offering an ex ante contract.

The paper is structured as follows: In the next section, we relate the problem to previous literature. Section 3 lays out the general model. Section 4 presents the results, first for ex post contracting, then for ex ante contracting, and finally compares ex ante with ex post contracting. Section 5 concludes. A characterization of the equilibria and the equilibrium contracts, and the proofs of lemmas, are relegated to the appendix.

2 Related Literature

Much of the previous literature has examined environments in which the principal and agent have symmetric information about the principal's technology. Gjesdal [5], in an agency model with ex ante contracting, and Grossman and Hart [6], in an agency model with ex post contracting, showed that when the principal and agent have symmetric information, the principal prefers to have a more informative technology. This arises from the fact that she is better able to control the action that the agent chooses and thereby implement any action at a lower cost than if she had a less informative technology.

Gjesdal [5] examined the value to the agency of acquiring additional information about both the unknown state of the world and the agent's action, i.e., the value of knowing better the information system that relates the agent's type or action to the outcomes. Under his assumptions, if one information system is a Blackwell transformation of another, then the principal prefers the latter to the former.[2] His model identifies two sources of marginal value for an information system: marginal

[2] A Blackwell transformation alters the more informative information system by multiplying it by a stochastic

insurance value through better risk-sharing and marginal incentive informativeness through making the action closer to first-best. In this paper, we show that when the agent does not know the principal's technology, the marginal insurance value is actually negative.

Maskin and Tirole [13] examined a model in which the principal has private information that affects the agent's expected utility. In such a principal-agent model with common values,[3] the agent receives his reservation utility but the principal may not attain her complete information payoff.

Another related paper is Inderst [8], in which he examines a principal-agent model with moral hazard and privately informed principal. In that paper, the principal can signal her information by the contract that she offers. Unlike in our model, the agent is risk neutral and the single-crossing property is satisfied. Inderst shows that the presence of private information distorts the contract and the action implemented, but the high type still earns more profit than the low type earns.

Chade and Silvers [1] examined a specific form of private information. They showed that in an agency model with moral hazard, when the principal has private information about the technology, there exist equilibria in which the principal with the more informative technology earns less profit than the one with the less informative technology, and there exist equilibria in which the agent receives more than his reservation utility.

Not only is existence and type of private information consequential, but also the timing of contracting has significant effects. Sobel [19] considered the situation in which the agent may acquire information about the state of the world prior to contracting, after contracting, or never. He showed that a risk neutral principal prefers to contract with an informed agent than with an uninformed agent.[4] By acquiring information, the agent implements a higher action but risk-sharing possibilities are reduced.

Kim [11] examined the timing of public information in a principal-agent model with moral hazard and showed that the principal prefers that information arrive after the agent has chosen his action, rather than before the agent chooses his action. Similarly, we show that the principal prefers receiving information later than sooner. Moreover, she realizes gains that are large enough to compensate the agent and make him better off. There are two key differences between Kim's model and ours. First, the information in Kim's model is knowledge about a random variable that

matrix, a matrix whose elements $r_{ij} \in (0,1)$ and such that $\sum_{i=1}^{N} r_{ij} = 1 \ \ \forall j$. The ijth element of R transforms the observed signal so that, the original information structure produces a signal s_j, implies that the transformed information structure produces the signal s_i with probability r_{ij}. It essentially garbles the signals, so that when an individual with the first information system observes one signal, another individual with the second, garbled, information system, observes that signal stochastically.

[3]A situation is said to have *common values* if the principal's private information enters the agent's expected utility function directly, such as the technology or disutility of effort. Whereas, the situation is said to be one of *private values* if the principal's private information does not enter the agent's expected utility function directly. For example, in the procurement of a public good, the government has private information about the cost – this does not affect the agent's (citizen's) expected utility function except indirectly, through the government's decision on whether to build the project or not. Another example is government procurement in which the government knows the value of the good to be supplied.

[4]Note that in his proof, this result may fail when more than two outcomes are possible.

affects the marginal utility of income; whereas, in our model, the information affects the returns to effort. Second, Kim considers situations in which information arrives after contracting and compares a situation in which information arrives before the agent chooses his action against one where information arrives after he chooses his action but prior to payment; whereas, we consider situations in which information arrives before the agent chooses his action and compare the effects of it arriving prior to contracting versus after contracting. Thus, in principal-agent models with moral hazard, along with Sobel's [19] results, it seems that generally, but not always, the principal prefers information to arrive later than sooner.

To see this contrast more clearly, in Kim's model, the agent receives his reservation utility regardless of when information arrives. The gains to the principal in ex ante contracting are due to the ability to design a contract that implements an action closer to first-best or that reduces the payment to the agent by equating the marginal costs of income across the different realizations of the random variable; however, these are outweighed by the costs of the agent adjusting his effort level because he also learns the realization of the random variable. In our model, the gains from ex ante contracting are due to the ability to insure against the realization of a noisy technology.

The existing literature therefore has not examined the consequences of different information structures and different timing of contracts in agency models with moral hazard. As Chiappori and Salanié [2][5] and Cohen [3] showed, and Schlesinger [17] argued,[6] in insurance markets, the insurer often has private information. Fluet [4] showed that as a company's fleet size increases, the equilibrium utilities approach first-best under ex post symmetric information. Together, these papers show that contracts and profits differ when insurers contract with new or small, versus experienced or large insurees. Our results show that symmetry versus asymmetry of information may explain these results.

3 Model

We consider a principal-agent model with moral hazard. The agent is a risk averse expected utility maximizer with additively separable vonNeumann-Morgenstern utility function over income and effort, given by $U(I, a) = V(I) - a$, with $V'(I) > 0, V''(I) < 0$; $\exists \underline{I}$ such that $lim_{I \downarrow \underline{I}} V'(I) = \infty$.

The agent chooses an action a_m from a set $\{a_1, \ldots, a_M\}$ where $0 < a_1 < a_2 < \ldots < a_M < \infty$ and $M \geq 2$. Through a stochastic process, the action chosen determines an outcome q_n from a set $\{q_1, \ldots, q_N\}$ where $0 < q_1 < q_2 < \ldots < q_N < \infty$ and $N \geq 2$.

Let $\pi_n(a_m)$ denote the conditional probability that the outcome is q_n given that the agent chose a_m. A *technology* is a Markov matrix whose elements are $\pi_n(a_m)$.

[5] See Chiappori and Salanié, page 73: "... the information at the company's disposal is extremely rich and that, in most cases, the asymmetry, if any, is in favor of the company."

[6] He stated that, compared to individual drivers, "insurance company actuaries will have a much better probability prediction for" the probability that a driver will experience an automobile accident within the next 10,000 miles of driving, and further that "the overall evidence shows a very uninformed population when it comes to insurance."

A risk neutral principal is endowed with one of two technologies, Π_1 or Π_0. Π_1 is more informative than Π_0 in the sense of Blackwell; i.e., a stochastic matrix, R, transforms Π_1 to Π_0 by $\Pi_0 = \Pi_1 R^T$. Let $\lambda \in [0,1]$ be the prior probability that the principal has Π_1. Define $\Pi_\lambda = \lambda\Pi_1 + (1-\lambda)\Pi_0$; each of its elements, $\pi_{\lambda n}(a_m)$, is the conditional probability that outcome q_n is realized given that the agent chose a_m and the beliefs about the principal's technology are λ. Π_λ denotes the principal who believes she has Π_1 with probability λ.

We assume that Π_1 and Π_0 both satisfy the monotone likelihood ratio property (MLRP) and convexity of the distribution function condition (CDFC). It is well known that together, these imply that the optimal perfect information contract is monotone in the outcome and the only incentive compatibility constraints that can bind are those that ensure the agent prefers not to choose a lower action (Salanié [16]).

MLRP states that the choice of a higher action increases the relative probability of a higher outcome compared to a lower outcome; formally, $\forall m' \leq m$, $\forall n' \leq n$, $\frac{\pi_{\lambda n}(a_{m'})}{\pi_{\lambda n'}(a_{m'})} \leq \frac{\pi_{\lambda n}(a_m)}{\pi_{\lambda n'}(a_m)}$.[7] Let $F(\tilde{n}, a_m) = \sum_{n=1}^{\tilde{n}} \pi_{\lambda n}(a_m)$ be the c.d.f. of Π_λ generated when the agent selects a_m.

CDFC states that for $i < j < k$, and for $\iota \in (0,1)$ such that $a_j = \iota a_i + (1-\iota)a_k$, $F(\tilde{n}, a_j) \leq \iota F(\tilde{n}, a_i) + (1-\iota)F(\tilde{n}, a_k)$. CDFC roughly implies that the returns to the action are stochastically decreasing.

Note that if Π_1 and Π_0 satisfy CDFC, then so too does Π_λ. However, MLRP does not necessarily carry forward, and so we make the additional assumption that Π_λ satisfies MLRP.

An ex post contract $I_\lambda = \{I_{\lambda 1}, \ldots, I_{\lambda N}\}$ is a specification of outcome-contingent payments from the principal to the agent, with $I_{\lambda n} \in \Re \; \forall n \in \{1, \ldots, N\}$. For clarity when comparing contracts that implement different actions, we write $I_\lambda(a_m)$ and $I_{\lambda n}(a_m)$ for the contract and wage, respectively.

$B_\lambda(a_m)$ is the benefit (revenue) for Π_λ from implementing a_m: $B_\lambda(a_m) = \sum_{n=1}^{N} \pi_{\lambda n}(a_m) q_n$. We assume revenue equivalence so that $B_1(a_m) = B_\lambda(a_m) = B_0(a_m)$.

$C_{\lambda_1}(I_{\lambda_2}(a_m))$ is the cost for Π_{λ_1} to implement a_m with I_{λ_2}: $C_{\lambda_1}(I_{\lambda_2}(a_m)) = \sum_{n=1}^{N} \pi_{\lambda_1 n}(a_m) I_{\lambda_2 n}$. Note that $\lambda_1 C_1(I_{\lambda_2}(a_m)) + (1-\lambda_1)C_0(I_{\lambda_2}(a_m)) = C_{\lambda_1}(I_{\lambda_2}(a_m))$.

An ex post contract is individually rational if it yields expected utility, given the agent's optimal effort level, that weakly exceeds his reservation utility, $\bar{U}$; i.e.,

$$\sum_{n=1}^{N} \pi_{\lambda n}(a_m) V(I_{\lambda n}) - a_m \geq \bar{U} \tag{1}$$

An ex post contract is incentive compatible if it induces the agent to choose the action that the principal wants to implement: $a_m \in \underset{a \in \{a_1, \ldots, a_M\}}{argmax} \sum_{n=1}^{N} \pi_{\lambda n}(a)\ V(I_n) - a$ or

$$\forall \tilde{m} \neq m \qquad \sum_{n=1}^{N} [\pi_{\lambda n}(a_m) - \pi_{\lambda n}(a_{\tilde{m}})] V(I_{\lambda n}(a_m)) \geq a_m - a_{\tilde{m}} \tag{2}$$

[7] Equivalently, this states that the likelihood of an outcome resulting from one action versus a lower action, is increasing in the outcome; i.e., $\frac{\pi_{\lambda n'}(a_m)}{\pi_{\lambda n'}(a_{m'})} \leq \frac{\pi_{\lambda n}(a_m)}{\pi_{\lambda n}(a_{m'})}$.

If the agent believes that the principal is Π_λ, denote the individual rationality and incentive compatibility constraints corresponding to implementing a_m by $IR(\lambda, a_m)$ and $IC(\lambda, a_m, a_{\tilde{m}})$, respectively.

If the principal implements $a_m > a_1$, the contract must satisfy both (1) and (2). If she implements a_1, she does so with the constant wage $\bar{I} = V^{-1}(\bar{U} + a_1)$.

The principal will observe a signal, z_k, of her technology. Let $Z = \{z_1, z_2\}$ be the signal space and ζ_{lk} the probability that signal z_k is sent when her technology is $\Pi_l, l \in \{0, 1\}$. λ is a common prior probability that the principal has Π_1; therefore, by Bayes' rule, $\lambda(z_k) = \frac{\lambda\zeta_{1k}}{\lambda\zeta_{1k}+(1-\lambda)\zeta_{0k}}$ is the probability that the principal has Π_1 conditional upon observing z_k.

An *information structure* is a Markov matrix

$$\zeta = \begin{bmatrix} \zeta_{01} & \zeta_{02} \\ \zeta_{11} & \zeta_{12} \end{bmatrix}$$

where $\zeta_{lk} \geq 0$, $\sum_k \zeta_{0k} = \sum_k \zeta_{1k} = 1$. The probability of observing z_k is then $prob(z_k) = \lambda\zeta_{1k} + (1-\lambda)\zeta_{0k}$.

The information structure determines the level of knowledge of the principal and the agent regarding her technology. By *null information*, we mean that the party (the principal or the agent) has received no information about the principal's technology and so the party's interim beliefs equal the prior beliefs. A player with null information is said to be *ignorant*. By *perfect information*, we mean that the party knows precisely the principal's technology, while *imperfect information* is where the party has received a signal that is imprecisely correlated with the principal's technology.

The specification of information symmetry determines the level of the agent's knowledge of the principal's technology and what is common knowledge. *Symmetric information* means that both the principal and the agent have the same knowledge about the principal's technology. *Asymmetric information* is where the principal knows more about her technology than the agent does. *Complete information* refers to the situation in which information is both perfect and symmetric.

The agent observes an event that contains the signal that the principal received. Let $t : Z \to S$, where $\#S \geq \#Z$, be an *information function.*

A particular information structure and an information function form an *environment.* We examine the following three environments[8]:

- *Complete Information*

  This environment is where $\zeta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and the agent's information function is one-to-one. Then, the principal and the agent have symmetric information. Each knows the principal's

[8] We do not examine other situations such as: symmetric null information – in which case the ex post and ex ante contracts are identical; symmetric imperfect information – in which case the comparisons are identical to complete information; and asymmetric perfect information where the agent has imperfect information – in which case the contracts are similar to asymmetric perfect information where the agent is ignorant. We also restrict attention to situations in which any private information favors the principal.

technology. This is the environment that Grossman and Hart [6] examined.

- *Asymmetric Perfect Information*

  This environment is where $\zeta = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and the agent's information function is constant. Then, the principal has perfect information of her technology while the agent is entirely ignorant, so that his interim beliefs equal his prior beliefs. This is the environment that Chade and Silvers [1] examined, except that we generalize here to more than two actions and more than two outcomes.

- *Asymmetric Imperfect Information*

  This environment is where $\zeta = \begin{bmatrix} \zeta_{01} & \zeta_{02} \\ \zeta_{11} & \zeta_{12} \end{bmatrix}$ and the agent's information function is constant. Then, the principal has imperfect information about her technology, while the agent is entirely ignorant, so that his interim beliefs equal his prior beliefs. Without loss of generality, we assume $\zeta_{11} > \zeta_{01}$ so that $\lambda(z_1) > \lambda > \lambda(z_2)$. If $\zeta_{01} = \zeta_{12} = 0$, then this environment reduces to *Asymmetric Perfect Information*, while if $\zeta_{01} = \zeta_{11}$, then it reduces to null, and thus symmetric, information.

The principal will implement an action for each of the possible signals she will receive. We call $\{a_m(z_1), a_m(z_2)\}$ the action profile and if $a_m(z_1) = a_m(z_2)$ then we say that the action profile is constant, else it is non-constant.

4 Results

We begin by describing the timing of the ex post contracting game and the principal's program, and then we show that the value of information for the principal makes the principal worse off but the agent may be better or worse off. In the next subsection, we describe the timing of the ex ante contracting game and the principal's program, and then we show that the value of information is positive for the principal. The last subsection then compares the two situations and shows that, by the Potential Pareto Criterion, ex ante contracting is superior to ex post contracting.

4.1 Ex Post Contracting

We examine Perfect Bayesian Equilibria (PBE) that can arise in the *Complete Information*, *Asymmetric Perfect Information*, and *Asymmetric Imperfect Information* environments. We compare the possible profits, utilities, and actions implemented.

The timing of the ex post contracting game is as follows:

1. Nature chooses an information structure and an information function;

2. Nature informs both the principal and the agent of these choices;

3. Nature chooses a technology;

4. Nature sends a signal to the principal according to the choices in 1;

5. the principal, having received z_k, updates her prior to her posterior beliefs about the technology she has; the agent, having observed the event that contains z_k as determined by his information function, updates his prior to his interim beliefs; and then the principal offers a contract to the agent;

6. the agent, having received the contract offer, updates his interim to his posterior beliefs, and then chooses whether to accept or reject; if the agent rejects, the game ends and he receives $\bar{U}$ while the principal receives 0; else

7. having accepted, he then chooses an action; and

8. Nature chooses an outcome according to the technology from 3 and the action choice from 7, and payoffs are made.

For each a_m, Π_λ solves the following program:

$$Min \sum_{n=1}^{N} \pi_{\lambda n}(a_m) I_{\lambda n} \quad s.t. \ (1) \ and \ (2) \tag{3}$$

This yields an ex post contract, $I_\lambda(a_m)$, and a cost to implement a_m. Π_λ then implements the action a_m that satisfies

$$a_m \in \underset{a \in \{a_1,\ldots,a_M\}}{argmax} \ B(a) - C_\lambda(I_\lambda(a)) \tag{4}$$

However, as the principal may have private information, it is possible for one type of principal to mimic another. $\Pi_{\lambda'}$ will not mimic Π_λ only if

$$B(a_{m'}) - C_{\lambda'}(I_{\lambda'}(a_{m'})) \geq B(a_m) - C_{\lambda'}(I_\lambda(a_m)) \tag{5}$$

Then, Π_λ solves the amended program:

$$\begin{aligned} &Min \ \sum_{n=1}^{N} \pi_{\lambda n}(a_m) I_{\lambda n} \\ &s.t. \ (1) \ and \ (2) \ and \ (5) \end{aligned} \tag{6}$$

The appendix contains a summary and characterization of the equilibrium contracts and consequent payoffs in the possible separating and pooling equilibria.

Let $\lambda \in [0, 1]$, $\mathbf{\Pi}$ be the set of technologies that satisfy MLRP and CDFC, and $\mathbf{R}$ be the set of stochastic matrices such that both Π_0 satisfies MLRP and CDFC, and Π_λ satisfies MLRP for all

$\lambda \in [0, 1]$. For a particular environment, a specific $\{\lambda, \Pi_1, R\}$, implies a set of equilibrium payoffs each for the principal and the agent.

Before stating our main result, we introduce notation and prior results about contracts and payoffs.

Let Y_f be the equilibrium payoff set for a player corresponding to one specification $\{\lambda, \Pi_1, R\}$, and y_f a particular equilibrium payoff in this set. Y_f is a real-valued set that need not be convex. Following Shannon [18], we have:

Definition 1 *Ranking of Sets*

Let Y_f and Y_g be two real-valued sets. Y_f is strong set order greater than *Y_g if $\forall y_f \in Y_f$ and $\forall y_g \in Y_g$, both $max(y_f, y_g) \in Y_f$ and $min(y_f, y_g) \in Y_g$.*

Y_g is completely lower than *Y_f if $\forall y_f \in Y_f$ and $\forall y_g \in Y_g$, $y_g \leq y_f$.*

Finally, Y_g is weakly lower than *Y_f if $\forall y_f \in Y_f$ and $\forall y_g \in Y_g$, either $max(y_f, y_g) \in Y_f$ or $min(y_f, y_g) \in Y_g$.*

MLRP and CDFC imply that the incentive compatibility constraints that can bind are $IC(\lambda, a_m, a_{\tilde{m}})$ where $a_{\tilde{m}} < a_m$.

In ex post contracting, it is well known that in order to implement the lowest action, the principal merely offers a flat wage, $\bar{I}$. Thus, a principal's profit from implementing this action is $B(a_1) - \bar{I}$.

Let I^*_λ denote the principal's optimal ex post contract that implements a_m in a PBE for Π_λ.

Because we assume that there is no natural separation, let $\hat{I}_\lambda$ represent the ex post contract that is least-cost for Π_λ among the set of ex post contracts that satisfy $IR(\lambda, a_m)$ and that $\Pi_{\lambda'}$ would not mimic where $\lambda' < \lambda$ and $\Pi_{\lambda'}$ is another possible type. $\hat{I}_\lambda$ clearly cannot satisfy the same incentive compatibility constraints with strict equality that I^*_λ satisfies. Let, $\tilde{M} = \{\tilde{m}_i : \hat{I}_\lambda$ satisfies $IC(\lambda, a_m, a_{\tilde{m}_i})$ with equality$\}$.

Lemma 1 *Grossman and Hart ([6], Proposition 13). Consider the* Complete Information *environment with ex post contracting, and let $\Pi_{\lambda'}$ be a Blackwell transformation of Π_λ. $\Pi_{\lambda'}$'s cost of her perfect information contract is greater than Π_λ's cost of her perfect information contract; i.e., $C_{\lambda'}(I^*_{\lambda'}(a_m)) > C_\lambda(I^*_\lambda(a_m))$.*

Chade and Silvers [1] showed that if the principal had private information about her technology, the agent would be worse off if information became symmetric and the principal would know that he learned her technology. Below, we show that the principal also prefers a less informative information structure if a more informative information structure were to become common knowledge in the following sense: If the principal has private, imperfect information about her technology, a Blackwell improvement in the information structure makes her worse off.

Chade and Silvers also showed that $\Pi_{\lambda(z_1)}$ can earn either more or less profit than $\Pi_{\lambda(z_2)}$ does in any separating equilibrium. Because we focus on the ex ante expected profit of the principal, the weighted average of the profits may increase even if the profits of each type decrease. This could happen if with the more informative information structure, the principal received the signal associated with the more profitable contract more often.

A Blackwell improvement in the information structure has two effects: it increases the posterior probability, conditional on observing z_1, that she has Π_1 – and similarly increases the posterior probability, conditional on observing z_2, that she has Π_0 – and changes the probability of receiving z_1 versus z_2. If the stochastic matrix relating ζ and ζ' is such that $r_{11} = 1 + r_{12}(1 - \frac{1}{prob(z_1|\zeta')})$, then $prob(z_1 \mid \zeta') = prob(z_1 \mid \zeta)$. For $r_{11} < 1 + r_{12}(1 - \frac{1}{prob(z_1|\zeta')})$, $prob(z_1 \mid \zeta') < prob(z_1 \mid \zeta)$. Because the inequality has two parameters, r_{11} and r_{12}, the sets of stochastic matrices that transform ζ' into ζ and yield $prob(z_1 \mid \zeta') < prob(z_1 \mid \zeta)$ or $prob(z_1 \mid \zeta') > prob(z_1 \mid \zeta)$ are nonempty.

Finally, note that the theorem shows that the cost of each contract rises, so that focusing only on the improvement in the informativeness of the information structure – i.e., requiring $r_{11} = 1 + r_{12}(1 - \frac{1}{prob(z_1|\zeta')})$ – yields the conclusion that the value of information for the principal is negative.

Theorem 1 *Negative Value of Information for the Principal When She Has Private Information*

Consider the Asymmetric Imperfect Information *environment. Let ζ' be less informative than ζ. The principal's equilibrium payoff set with ζ' is strong set order greater than that with ζ if either condition below holds:*

1. *The profit for $\Pi_{\lambda(z_1|\zeta)}$ from offering $\hat{I}_{\lambda(z_1|\zeta)}$ is greater than the profit for $\Pi_{\lambda(z_2|\zeta)}$ from offering $I^*_{\lambda(z_2|\zeta)}$, and the ex ante probability of receiving z_1 with ζ' is at least that with ζ; or*

2. *The profit for $\Pi_{\lambda(z_1|\zeta)}$ from offering $\hat{I}_{\lambda(z_1|\zeta)}$ is less than the profit for $\Pi_{\lambda(z_2|\zeta)}$ from offering $I^*_{\lambda(z_2|\zeta)}$, and the ex ante probability of receiving z_1 with ζ' is no greater than that with ζ;*

Proof:

The set of pooling equilibria with ζ and with ζ' are identical.

Consider the separating equilibria. Without loss of generality, in order to focus on the costs only, assume that the principal implements a_m given z_1 or z_2 in ζ' and in ζ. We will show that the expected cost of each contract is lower with ζ' than with ζ. Combined with the assumption that the principal will implement the more profitable contract at least as often, this implies that her expected cost is lower with ζ'. Therefore, for any separating equilibrium, if the principal were to implement a non-constant action profile when the information structure is ζ, then she could implement that identical action profile when the information structure is ζ', and her expected profit would increase.

Consider first the case in which $C_{\lambda(z_1|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)}) > C_{\lambda(z_2|\zeta)}(I^*_{\lambda(z_2|\zeta)})$; i.e., the principal with the more informative technology implements a_m at a higher cost than does the principal with the less informative technology (Chade and Silvers [1] showed that this is possible). After the Blackwell transformation, $\lambda(z_2|\zeta') > \lambda(z_2|\zeta)$, which implies by Lemma 1, that $C_{\lambda(z_2|\zeta)}(I^*_{\lambda(z_2|\zeta)}) > C_{\lambda(z_2|\zeta')}(I^*_{\lambda(z_2|\zeta')})$. Because the technologies, $\Pi_{\lambda(z_2|\zeta)}, \Pi_{\lambda(z_2|\zeta')}, \Pi_{\lambda(z_1|\zeta')}, \Pi_{\lambda(z_1|\zeta)}$ are convex combinations of Π_1 and Π_0 with $\lambda(z_2|\zeta) < \lambda(z_2|\zeta') < \lambda(z_1|\zeta') < \lambda(z_1|\zeta)$, $C_{\lambda(z_1|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')}) > C_{\lambda(z_2|\zeta')}(I^*_{\lambda(z_2|\zeta')})$.

Consider the difference in the expected costs of a contract for the principal when she receives z_1 versus z_2 . The Blackwell transformation decreases this cost difference; i.e., $C_{\lambda(z_1|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')}) - C_{\lambda(z_2|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')}) < C_{\lambda(z_1|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)}) - C_{\lambda(z_2|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)})$. That is,

$$C_{\lambda(z_1|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)}) - C_{\lambda(z_1|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')}) > \\ C_{\lambda(z_2|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)}) - C_{\lambda(z_2|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')}) = \\ C_{\lambda(z_2|\zeta)}(I^*_{\lambda(z_2|\zeta)}) - C_{\lambda(z_2|\zeta')}(I^*_{\lambda(z_2|\zeta')}) > 0$$

Therefore, $C_{\lambda(z_1|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)}) > C_{\lambda(z_1|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')})$, so that the contracts that the principal offers both given z_1 and given z_2 are each less expensive with ζ' than with ζ. Since $prob(z_1|\zeta') \leq prob(z_1|\zeta)$, the expected cost decreases.

Consider the other case in which $C_{\lambda(z_1|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)}) < C_{\lambda(z_2|\zeta)}(I^*_{\lambda(z_2|\zeta)}) = C_{\lambda(z_2|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)})$; i.e., the principal with the more informative technology implements a_m at a lower cost than does the principal with the less informative technology. As in the other case, after the Blackwell transformation, $C_{\lambda(z_2|\zeta')}(I^*_{\lambda(z_2|\zeta')}) < C_{\lambda(z_2|\zeta)}(I^*_{\lambda(z_2|\zeta)})$ and because $prob(z_1|\zeta') \geq prob(z_1|\zeta)$, in order to guarantee that the principal's ex ante expected profit is greater with ζ', it suffices to show that $C_{\lambda(z_1|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')}) < C_{\lambda(z_1|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)})$.

First, note that both $\Pi_{\lambda(z_1|\zeta)}$ and $\Pi_{\lambda(z_2|\zeta)}$ prefer the separating contract with ζ' to the separating contract with ζ; i.e., $C_{\lambda(z_2|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)}) > C_{\lambda(z_2|\zeta)}(\hat{I}_{\lambda(z_1|\zeta')})$ and $C_{\lambda(z_1|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)}) > C_{\lambda(z_1|\zeta)}(\hat{I}_{\lambda(z_1|\zeta')})$.

The Blackwell transformation decreases the cost difference for the principals who receive either signal. That is,

$$C_{\lambda(z_1|\zeta)}(I^*_{\lambda(z_1|\zeta)}) - C_{\lambda(z_1|\zeta')}(I^*_{\lambda(z_1|\zeta')}) < C_{\lambda(z_2|\zeta)}(I^*_{\lambda(z_2|\zeta)}) - C_{\lambda(z_2|\zeta')}(I^*_{\lambda(z_2|\zeta')}) = \\ C_{\lambda(z_2|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)}) - C_{\lambda(z_2|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')})$$

Also, $C_{\lambda(z_2|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)}) - C_{\lambda(z_1|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)}) > C_{\lambda(z_2|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')}) - C_{\lambda(z_1|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')})$ and $C_{\lambda(z_2|\zeta)}(\hat{I}_{\lambda(z_1|\zeta')}) - C_{\lambda(z_1|\zeta)}(\hat{I}_{\lambda(z_1|\zeta')}) > C_{\lambda(z_2|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')}) - C_{\lambda(z_1|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')})$.

With either information structure, $\Pi_{\lambda(z_1|\cdot)}$'s cost is bounded by her cost of $I^*_{\lambda(z_2|\cdot)}$ because she cannot mimic $\Pi_{\lambda(z_2|\cdot)}$ in a PBE; these contracts decreases in cost. $\Pi_{\lambda(z_1|\zeta)}$ has a lower cost of $\hat{I}_{\lambda(z_1|\zeta)}$ than of $I^*_{\lambda(z_2|\zeta)}$ and $\Pi_{\lambda(z_1|\zeta')}$ has a lower cost of $\hat{I}_{\lambda(z_1|\zeta')}$ than of $I^*_{\lambda(z_2|\zeta')}$.

The weighted average cost of the symmetric information contracts is declining to I^*_λ with Blackwell transformations in the information structure and the costs of the contracts are continuous in the technologies.

Additionally, $\Pi_{\lambda(z_1|\zeta')}$ has a lower cost of $\hat{I}_{\lambda(z_1|\zeta')}$ than does $\Pi_{\lambda(z_2|\zeta')}$ because both $\lambda(z_1|\zeta) > \lambda(z_1|\zeta') > \lambda(z_2|\zeta') > \lambda(z_2|\zeta)$ and $C_{\lambda(z_1|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)}) < C_{\lambda(z_2|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)})$.

Finally, observe that the gains from mimicking that $\Pi_{\lambda(z_2|\zeta')}$ would realize are smaller than those that $\Pi_{\lambda(z_2|\zeta)}$ would realize; i.e., $C_{\lambda(z_2|\zeta')}(I^*_{\lambda(z_2|\zeta')}) - C_{\lambda(z_2|\zeta')}(I^*_{\lambda(z_1|\zeta')}) < C_{\lambda(z_2|\zeta)}(I^*_{\lambda(z_2|\zeta)}) - C_{\lambda(z_2|\zeta)}(I^*_{\lambda(z_1|\zeta)})$. That is, $\hat{I}_{\lambda(z_1|\zeta')}$ satisfies a relatively relaxed constraint (5) compared to $\hat{I}_{\lambda(z_1|\zeta)}$.

Since $\Pi_{\lambda(z_1|\zeta)}$ has a lower cost of the separating contract than $\Pi_{\lambda(z_2|\zeta)}$ does, the separating contract with ζ' costs even less; i.e., $C_{\lambda(z_1|\zeta')}(\hat{I}_{\lambda(z_1|\zeta')}) < C_{\lambda(z_1|\zeta)}(\hat{I}_{\lambda(z_1|\zeta)})$.

The separating contract with ζ' must lie on an isocost set that is below the isocost set with ζ, the cost for the principal who receives z_1 for any such contract is less than both the costs for her to mimic the principal who receives z_2 and the costs of $\Pi_{\lambda(z_2|\cdot)}$, and the cost differences have diminished. Because $prob(z_1|\zeta') \geq prob(z_1|\zeta)$, the expected cost has increased. This completes the second case when the principal implements the same action for either signal.

Finally, for either case, let the principal implement a_m given z_1 but $a_{m'} \neq a_m$ given z_2 when the information structure is ζ. Suppose that she earns greater profit if she receives z_1 than if she receives z_2. She can still implement the same action profile when the information structure is ζ'. If with ζ', the ex ante probability of receiving z_1 has not decreased, then her profit is greater with the less informative information structure. Her profit is greater for either signal that she receives and she implements the more profitable contract at least as often.

If she earns greater profit if she receives z_2 than if she receives z_1 when the information structure is ζ, then, if the ex ante probability of receiving z_1 has not increased, then her profit is greater with the less informative information structure. ■

There are three differences that the Blackwell transformation generates: it raises the initial cost of the symmetric information contract from which the principal who receives z_1 alters wages in order to separate from the principal who receives z_2; it raises the signaling cost – the marginal cost that the principal who receives z_1 incurs when she increases the other principal's cost one dollar; and it reduces the amount of signaling required – the gains that the principal who receives z_2 would realize if she were able to mimic.

The Blackwell transformation restricts the incentive compatibility constraint and relaxes the individual rationality constraint. The relevant comparison for whether the principal prefers a more informative information structure rests in part on the expected costs of two separating contracts for two types – $\hat{I}_{\lambda(z_1|\zeta')}$ for $\Pi_{\lambda(z_1|\zeta')}$ and $\hat{I}_{\lambda(z_1|\zeta)}$ for $\Pi_{\lambda(z_1|\zeta)}$. When weighted by the relative probability, the former type has a lower cost for the former contract than does the latter type for the latter contract. This follows from the relative gradients of the iso-cost surfaces for the four types of principal – $\Pi_{\lambda(z_1|\zeta')}, \Pi_{\lambda(z_1|\zeta)}, \Pi_{\lambda(z_2|\zeta')}, \Pi_{\lambda(z_2|\zeta)}$ – and the locations of the separating contracts in $N - space$.

Considering the choice of an information structure, a principal would only want a more informative information structure about her technology if the information structure makes her more likely to receive the signal that induces her to offer a contract that yields greater profit; however, even this is not sufficient for her to prefer the more informative information structure since each contract is also more expensive. This applies even if she were to implement a different action profile with the more informative information structure, since she could still implement that identical action profile with the less informative information structure.

Although the principal sometimes prefers a less informative information structure, the agent may gain or lose from the principal having a more informative information structure, as the example shows.

Example 1 *Positive or Negative Value of the Principal Having a More Informative Information Structure, for the Agent*

Let the principal have asymmetric imperfect information and consider the two technologies: $\Pi_1 = \begin{bmatrix} 0.6 & 0.3 & 0.1 \\ 0.3 & 0.3 & 0.4 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$ and $\Pi_0 = \begin{bmatrix} 0.6 & 0.25 & 0.15 \\ 0.48 & 0.31 & 0.21 \\ 0.4 & 0.35 & 0.25 \end{bmatrix}$. They are related by the stochastic matrix $R = \begin{bmatrix} 0.7 & 0.5 & 0.3 \\ 0.2 & 0.3 & 0.4 \\ 0.1 & 0.2 & 0.3 \end{bmatrix}$.

The principal's initial information structure is: $\zeta'' = \begin{bmatrix} 0.14 & 0.86 \\ 0.84 & 0.16 \end{bmatrix}$.

The agent has square root utility, reservation utility $\bar{U} = 50$, and $a \in \{0, 2, 4\}$. $\lambda = .55$.

$\lambda(z_1) = 0.88$ and $\lambda(z_2) = 0.185263$. The principal implements a_2 with the following ex post contracts: $\hat{I}_{\lambda(z_1)} = \{42.1492, 66.5035, 48.8772\}$ and $I^*_{\lambda(z_2)} = \{45.4547, 55.0244, 60.122\}$. The agent's expected utilities from each contract are 52.0225 and 52, yielding an ex ante expected utility of 52.0118.

For $R_l = \begin{bmatrix} 0.0161644 & 0.975068 \\ 0.983836 & 0.0249315 \end{bmatrix}$, $\zeta' = \zeta''(R_l^T)^{-1}$ is a Blackwell improvement yielding the new information structure $\zeta' = \begin{bmatrix} 0.12 & 0.88 \\ 0.85 & 0.15 \end{bmatrix}$. $\lambda(z_1) = 0.896453$ and $\lambda(z_2) = 0.172414$. The principal implements a_2 with the following ex post contracts: $\hat{I}_{\lambda(z_1)} = \{42.0022, 66.797, 48.7112\}$ and $I^*_{\lambda(z_2)} = \{45.348, 55.185, 60.2578\}$. The agent's expected utilities from each contract are now 52.0179 and 52, yielding an ex ante expected utility of 52.0093, which is less than his expected utility before the Blackwell improvement.

A further Blackwell improvement such that $\zeta' = \zeta R_g^T$ where $R_g = \begin{bmatrix} 0.0925301 & 0.935904 \\ 0.90747 & 0.0640964 \end{bmatrix}$ yields the new information structure $\zeta = \begin{bmatrix} 0.09 & 0.91 \\ 0.92 & 0.08 \end{bmatrix}$. $\lambda(z_1) = 0.925892$ and $\lambda(z_2) = 0.0970232$. The principal implements a_2 with the following ex post contracts: $\hat{I}_{\lambda(z_1)} = \{41.513, 67.9323, 48.1836\}$ and $I^*_{\lambda(z_2)} = \{44.6361, 56.344, 61.0338\}$. The agent's expected utilities from each contract are now 52.0327 and 52, yielding an ex ante expected utility of 52.0178, which is greater than his expected utility before the Blackwell improvement.

Observe that the principal's expected cost of the original pair of ex post contracts is 2774.92; her expected costs of the ex post contracts after the R_l and R_h transformations are 2777.02 and 2791.95. The principal is worse off after each Blackwell improvement. ■

The example may seem to contradict the conclusion of Theorem 1 since a Blackwell transformation raises the principal's profit but may either lower or raise the agent's expected utility. If the agent were to receive the same utility but the wages were to compress, the principal's cost would decline since there is less ex ante risk. If the agent's utility were to decline, then the principal's cost would also certainly decline; whereas, if the agent's utility were to increase, then the principal's cost increases. However, this increase need not outweigh the decrease from the compression of the wages. Thus, the two results are not contradictory.

In the separating equilibria, the agent receives $\bar{U}$ from the ex post contract $I^*_{\lambda(z_2)}$ but more than this from $\hat{I}_{\lambda(z_1)}$. In the example, the agent's expected utility from the separating contract is not monotonic with the Blackwell improvements, falling from 52.0225 to 52.0179 and then rising to 52.0327. While the probabilities of receiving the contract that cedes rents also are not monotonic, the difference between the effects on the principal and the agent is that the agent may get less utility from the separating contract, even if she were to receive it more often; whereas, the principal's cost of the separating contract is increasing.

A Blackwell transformation in the information structure has three ambiguous effects. First, it alters the probabilities of receiving each ex post contract and thus the ex ante expected utility of the equilibrium. A Blackwell transformation decreases ζ_{11} and increases ζ_{01}, but the effect on $prob(z_1) = \lambda\zeta_{11} + (1-\lambda)\zeta_{01}$ is ambiguous. Thus, the agent may receive the contract that cedes him rents more or less often.

Second, since $\lambda(z_1)$ decreases toward λ, the agent's conditional expected utility of $\hat{I}_{\lambda(z_1)}$ may increase or decrease depending upon whether this ex post contract is monotonic or not and whether the distribution generated by $\pi_1(a_m)$ first-order stochastically dominates that generated by $\pi_0(a_m)$, or vice versa. As $\Pi_{\lambda(z_2)}$ is a convex combination of these two technologies, it is possible that either first-order stochastically dominates the other; if the ex post contract is monotonically increasing, then a shift of probability from lower to higher outcomes increases the expected utility.

Finally, by altering both $\lambda(z_1)$ and $\lambda(z_2)$, the separating ex post contract itself must be changed. An increase in $\lambda(z_2)$ to $\lambda(z_{2'})$ makes $\Pi_{\lambda(z_2)}$ less willing to mimic since the cost of $I^*_{\lambda(z_{2'})}$ decreases, while that of $I^*_{\lambda(z_{1'})}$ increases. Thus, $\Pi_{\lambda(z_{1'})}$ needs to alter the ex post contract less. These changes may increase the agent's utility for a given $I_{\lambda(z_1)}$ or may even decrease it.

4.2 Ex Ante Contracting

With ex ante contracting, the timing is altered and consequently, when the principal will have private information, there are new incentive compatibility constraints. Recall the timing of the ex post contracting, wherein the principal offers the agent the contract *after* Nature sends the signal to the principal about her technology, and the agent both knows the chosen information structure and observes the event associated with that signal before accepting or rejecting the contract. In ex ante contracting, the principal offers a contract *prior* to Nature sending the signal about her technology, and thus prior to the agent observing the associated event. The agent makes his acceptance/rejection decision of the ex ante contract at this point, and if he accepts, only then does Nature send the signal to the principal – at which time the agent observes the associated event. Then, the principal announces, not necessarily truthfully, her type and the agent then selects his action.

The timing of the ex ante contracting game is as follows:

1. Nature chooses an information structure and an information function;
2. Nature informs both the principal and the agent of these choices;
3. Nature chooses a technology;
4. the principal offers an ex ante contract to the agent that specifies a payment contingent upon both the type that the principal will announce and the outcome;
5. the agent chooses whether to accept or reject; if the agent rejects, the game ends and he receives $\bar{U}$ while the principal receives 0; else,
6. Nature sends a signal to the principal according to the choice in 1;
7. the principal, having received z_k, updates her prior to her posterior beliefs about the technology she has; the agent, having observed the event that contains z_k as determined by his information function, updates his prior to his interim beliefs;
8. the principal announces to the agent a type $\Pi_{\lambda(z_k)}$;
9. the agent, having heard the principal's announcement, updates his interim to his posterior beliefs;
10. the agent chooses an action; and
11. Nature chooses an outcome according to the technology from 3 and the action choice from 10, and payoffs are made.

For a given k, $I_{\lambda(z_k)} = \{I_{\lambda(z_k)1}, \ldots, I_{\lambda(z_k)N}\}$ is the *announcement-contingent contract* that corresponds to the type announced. $I = \{I_{\lambda(z_1)}, I_{\lambda(z_2)}\} = \{I_{\lambda(z_1)1}, \ldots, I_{\lambda(z_1)N}, I_{\lambda(z_2)1}, \ldots, I_{\lambda(z_2)N}\}$ is an ex ante contract where $I_{\lambda(z_k)n}$ is the wage if the principal announces $\Pi_{\lambda(z_k)}$ and the outcome is q_n.

Consequently, the offer of the ex ante contract does not inform the agent about the principal's type, and what were no-mimic constraints are now incentive compatibility constraints for the principal. The ex ante contract is incentive compatible for the principal if, for each z_k, the principal has the incentive to truthfully announce her type. By making the acceptance/rejection decision prior to learning the principal's type, the various announcement-contingent contracts need not each yield expected utility $\bar{U}$, but rather the ex ante contract needs to satisfy individual rationality in expectation.[9] In designing it, the principal can trade off lower utility (and cost) from one report

[9]We show in Lemma 2 that the agent receives expected utility exactly equal to $\bar{U}$.

for higher utility (and cost) from another report.

A PBE requires that the principal report her type truthfully. No $\Pi_{\lambda(z_{k'})}$ reports $\Pi_{\lambda(z_k)}$ if and only if

$$B(a_{m_{\lambda(z_{k'})}}) - C_{\lambda(z_{k'})}(I_{\lambda(z_{k'})}(a_{m_{\lambda(z_{k'})}})) \geq B(a_{m_{\lambda(z_k)}}) - C_{\lambda(z_{k'})}(I_{\lambda(z_k)}(a_{m_{\lambda(z_k)}}))$$

Denote this constraint by $PIC_{\lambda(z_k)\lambda(z_{k'})}$.

Recall that $prob(z_k) = \lambda\zeta_{1k} + (1-\lambda)\zeta_{0k}$. The principal's program is now to select the optimal $I_{\lambda(z_k)n}$ for each possible action profile:

$$\begin{aligned} Min \quad & \sum_{k=1}^{2} prob(z_k) \sum_{n=1}^{N} \pi_{\lambda(z_k)n}(a_m(z_k)) I_{\lambda(z_k)n} \\ s.t. \quad & \sum_{k=1}^{2} prob(z_k) \sum_{n=1}^{N} \pi_{\lambda(z_k)n}(a_m(z_k)) V(I_{\lambda(z_k)n}) - a_m(z_k) \geq \bar{U} \\ & IC(\lambda(z_k), a_m, a_{\tilde{m}}) \quad \forall k \quad \forall a_{\tilde{m}} \neq a_m \\ & PIC_{\lambda(z_k)\lambda(z_{k'})} \quad \forall k \quad \forall k' \neq k \end{aligned} \tag{7}$$

and then to select the action profile that yields the greatest profit. Let $a_m^*(z_k)$ denote the action that the principal implements in the least-cost action profile if both she receives z_k and each announcement-contingent contract yields the agent $\bar{U}$. Let $I^*_{\lambda(z_k)}(a_m^*(z_k))$ denote the optimal announcement-contingent contract that has expected utility $\bar{U}$.

However, since the individual rationality constraint applies to the contract and not each announcement-contingent contract, the expected utility of each announcement-contingent contract need not equal $\bar{U}$. At the optimal ex ante solution, the principal may *choose to* provide different expected utilities for the different signals, and so may implement a different action profile. Let the optimal action profile implemented be denoted by $\{a_m^{**}(z_1), a_m^{**}(z_2)\}$ and, where $\lambda(z_k)$ is the agent's belief that the principal has Π_1, let $I^{**}_{\lambda(z_k)}(a_m^{**}(z_k))$ denote the optimal announcement-contingent contract. Let $u_k^{**} = \sum_{n=1}^{N} \pi_{\lambda(z_k)n}(a_m^{**}(z_k)) V(I^{**}_{\lambda(z_k)n}) - a_m^{**}(z_k)$ denote the agent's expected utility from the announcement-contingent contract $I^{**}_{\lambda(z_k)}(a_m^{**}(z_k))$ if the principal receives z_k and announces truthfully; and let $c^{**}_{k'k} = \sum_{n=1}^{N} \pi_{\lambda(z_{k'})n}(a_m^{**}(z_k)) I^{**}_{\lambda(z_k)n}(a_m^{**}(z_k))$ denote $\Pi_{\lambda(z_{k'})}$'s expected cost if she offers $I^{**}_{\lambda(z_k)}(a_m^{**}(z_k))$.

Because the ex ante contract would not be renegotiated, the solution to this program is equivalent to the solutions to the following programs, for each k:

$$
\begin{aligned}
&Min \;\; prob(z_k)\sum_{n=1}^{N}\pi_{\lambda(z_k)n}(a_m^{**}(z_k))I_{\lambda(z_k)n} \\
&s.t. \;\; \sum_{n=1}^{N}\pi_{\lambda(z_k)n}(a_m^{**}(z_k))V(I_{\lambda(z_k)n}) - a_m^{**}(z_k) \geq u_k^{**} \\
&\quad\;\; IC(\lambda(z_k), a_m, a_{\tilde{m}}) \quad \forall a_{\tilde{m}} \neq a_m \\
&\quad\;\; B(a^{**}_{m_{\lambda(z_{k'})}}) - C_{\lambda(z_{k'})}(I_{\lambda(z_{k'})}(a^{**}_{m_{\lambda(z_{k'})}})) \geq B(a^{**}_{m_{\lambda(z_k)}}) - c^{**}_{k'k} \quad \forall k' \neq k
\end{aligned}
\tag{8}
$$

This can be seen by forming the Lagrangians from each program for all possible z_k and adding them together. The value of the Lagrangian is the same as the value of the Lagrangian in the initial program (7). Moreover, if μ_k is the Lagrange multiplier on the individual rationality constraint in (8) and μ is the Lagrange multiplier on the individual rationality constraint in (7), then $\frac{\mu_k}{prob(z_k)} = \mu$.

Because the principal solves a minimization problem and, at the time of contract offer, she has no private information, the agent cannot hold any beliefs about the principal's type and so the ex ante contract yields unique payoffs. It is possible that a principal is indifferent between two contracts, but then they would yield her the same profit and as Lemma 2 below shows, the agent is also indifferent.

Let $I_\lambda^{**}(a_m)$ denote the announcement-contingent contract that satisfies (2) and yields the agent utility u_k^{**} if the agent's beliefs are λ, and $\hat{\hat{I}}_\lambda(a_m)$ denote the announcement-contingent contract that satisfies (2) and yields the agent utility u_k^{**} if the agent's beliefs are λ and also satisfies $PIC_{\lambda(z_k)\lambda(z_{k'})}$ for $\lambda(z_k) > \lambda(z_{k'})$ and for each $k' \neq k$.

Unless the principal has null information, the principal may be able to offer an ex ante contract that is pooling (the two announcement-contingent contracts are the same) or else is separating. The appendix summarizes and characterizes the equilibrium contracts and consequent payoffs.

In a private values setting, as Maskin and Tirole [12] showed, the principal can always offer the ex ante contract that comprises the contracts that would have been offered in ex post contracting and so can always do at least as well. However, the current setting is common values, and in a subsequent paper, Maskin and Tirole [13] showed that the agent may hold pessimistic beliefs that prevent the principal from realizing her full information payoff. That is, the principal may do worse when she has private or secret information than when information is symmetric. In our current setting, the principal offers an ex ante contract prior to obtaining any information about her technology, and so the agent cannot hold pessimistic beliefs upon receiving the ex ante contract.

We begin by showing that the agent is not better off with ex ante contracting in any environment since if an ex ante contract ever yielded more than $\bar{U}$, the principal could lower the wage associated with the minimum outcome in each announcement-contingent contract in such a way as to maintain incentive compatibility of the principal and also lower her costs, while being individually rational.

Throughout this section, we assume that the minimum wage from each announcement-contingent contract is strictly greater than $\underline{I}$.

Lemma 2 *Agent Receives $\bar{U}$*

Suppose that there is ex ante contracting. For any environment, every equilibrium ex ante contract yields the agent exactly $\bar{U}$.

Proof:

The program given by (8) for each possible signal shows that as long as for each z_k that occurs with positive probability, the $\frac{\mu_k}{prob(z_k)}$ are not equal to each other, then the principal can adjust an announcement-contingent contract to reduce her cost and reduce the agent's utility. This fraction, equal to μ at the optimal solution to (7), is the marginal cost of utility provision at the current ex ante contract. If the agent receives more than $\bar{U}$ from some ex ante contract, then the principal can adjust one or more announcement-contingent contracts by lowering the wages, and thereby the expected utility from that announcement-contingent contract and thus of the ex ante contract. This is possible as long as the minimum wage associated with each announcement-contingent contract is greater than $\underline{I}$. ■

Suppose that one announcement-contingent contract, $I_{\lambda(z_k)}$, had a minimum wage equal to $\underline{I}$. Then this announcement-contingent contract could provide more than $\bar{U}$ at its optimum because in order to satisfy incentive compatibility, the wage structure yields an expected utility greater than $\bar{U}$. The agent may still receive only $\bar{U}$, however, if other announcement-contingent contract(s) can be adjusted to provide less than $\bar{U}$. Only if for these other announcement-contingent contracts, the minimum wage also equals $\underline{I}$ or if $PIC_{\lambda(z_k)\lambda(z_{k'})}$ binds, would the agent receive more than $\bar{U}$ from the equilibrium ex ante contract.

Intuitively, contracting in agency models with moral hazard is utility provision subject to the agent's incentive compatibility constraint. The principal wants to minimize the cost to provide utility; in an ex ante contracting environment, the principal compares the marginal costs of utility provision, $\frac{\mu_k}{prob(z_k)}$, for each announcement-contingent contract. That with the lowest value is adjusted so as to provide more utility while that with the highest value is adjusted so as to provide less utility. The agent's expected utility of the ex ante contract is $prob(z_1)u_1 + prob(z_2)u_2$.

In ex post contracting, the principal would not voluntarily acquire a more informative information structure when the agent is ignorant unless it would result in her implementing the more profitable contract more often. In ex ante contracting, however, the principal prefers a more informative information structure, as the next theorem demonstrates.

Theorem 2 *Nonnegative Value of Information for the Principal When She Has Private Imperfect Information*

Consider the Asymmetric Imperfect Information *environment. Let ζ be a Blackwell improvement of ζ'; i.e., for some stochastic matrix R, $\zeta' = \zeta R^T$. If the principal were to implement a_m given either z_1 or z_2 with ζ', then her profit with ζ is weakly greater than that with ζ'.*

Proof:

Let ζ' be the initial information structure. As the principal implements a_m given z_1 or z_2 with ζ', $\{I^*_\lambda, I^*_\lambda\}$ is the equilibrium ex ante contract since this is the least-expensive contract that implements a_m for either signal and provides the agent $\bar{U}$.

Let ζ be a Blackwell improvement of ζ'. If with ζ, the principal implements the same action profile, then she does so with $\{I^*_\lambda, I^*_\lambda\}$ and earns the same profit. If she implements a different action profile, then her profit is:

$$prob(z_1)\Big(B(a_{m_{\lambda(z_1)}}) - C_{\lambda(z_1)}(\hat{I}_{\lambda(z_1)}(a_{m_{\lambda(z_1)}}))\Big) + \\ prob(z_2)\Big(B(a_{m_{\lambda(z_2)}}) - C_{\lambda(z_2)}(I^*_{\lambda(z_2)}(a_{m_{\lambda(z_2)}}))\Big) > B(a_m) - C_\lambda(I^*_\lambda(a_m))$$

She is necessarily better off since the same ex ante contract is possible. ■

If the principal implemented different actions given z_1 versus z_2 with ζ', then a Blackwell improvement may make her worse off. This follows because a Blackwell transformation of the information structure increases ζ_{11} and decreases ζ_{01}; therefore, $\lambda(z_1|\zeta') - \lambda(z_2|\zeta') < \lambda(z_1|\zeta) - \lambda(z_2|\zeta)$. If there were no principal incentive compatibility constraints to satisfy, then the cost may increase. The principal incentive compatibility constraints become more difficult to satisfy after the Blackwell improvement since $\Pi_{\lambda(z_2)}$ would have more to gain, and therefore would be more likely to mimic.

Intuitively, a decrease in $\lambda(z_2)$ increases the marginal cost of utility provision from the announcement-contingent contract corresponding to z_2. If there were no possibility of $\Pi_{\lambda(z_2)}$ selecting $I_{\lambda(z_1)}$, then the marginal cost of utility provision from $I_{\lambda(z_1)}$ would decrease, but the average would still increase. However, because the principal's incentive compatibility constraint is more difficult to satisfy, the marginal cost of utility provision decreases less, or may even increase. However, an increase in $\lambda(z_1)$ and a decrease in $\lambda(z_2)$ also each lower the signaling cost, which is given by the ratio $\frac{\pi_{\lambda(z_1)n}(a_m)}{\pi_{\lambda(z_2)n}(a_m)}$.

4.3 Ex Ante vs. Ex Post Contracting

The results from ex ante contracting show that the principal is always at least as well off with ex ante contracting as with ex post contracting. In contrast to Maskin and Tirole [13], because the principal offers the ex ante contract before becoming informed, she can offer as an ex ante contract the same announcement-contingent contracts that would obtain in an ex post contracting equilibrium. However, because she can transfer utility from one announcement-contingent contract with a lower marginal cost of utility provision to another announcement-contingent contract, she can do better.

The agent, on the other hand, is never better off with ex ante contracting. In ex ante contracting, he always gets only his reservation utility; whereas, in ex post contracting, when the principal has private information, the agent can earn rents as the types of principal use the ex post contracts to signal, and often in such a manner that the ex post contract makes the agent better off.

Chade and Silvers [1] showed that the agent earns strictly more than his reservation utility from the least-cost separating contract. We begin by extending this result from the two-action-two-outcome case to the M-action-N-outcome case. For ease of exposition, this proof appears in the appendix.

Lemma 3 *Consider the* Asymmetric Perfect Information *environment with ex post contracting. Assume $\lambda > \lambda'$. Let Π_λ implement a_m and $\Pi_{\lambda'}$ implement $a_{m'}$, where $a_m \geq a_{m'}$ and suppose $a_m > a_1$. The least-cost separating ex post contract for Π_λ yields the agent strictly more than his reservation utility.*

Proof: See Appendix 6.2.

Lemma 4 *Grossman and Hart ([6], Proposition 2). Consider the* Complete Information *environment with ex post contracting. The agent receives expected utility exactly $\bar{U}$; i.e., I^*_λ satisfies individual rationality exactly.*

Theorem 3 *Ex Ante Contracting Is Superior to Ex Post Contracting Using the Potential Pareto Criterion*

Assume that the minimum wage from each announcement-contingent contract is strictly greater than $\underline{I}$.

1. *Suppose the principal has private information. The agent's equilibrium payoff set with ex post contracting is strong set order greater than his equilibrium payoff in the same environments with ex ante contracting.*

2. *Suppose that information is complete. The agent's equilibrium payoff sets are identical and degenerate.*

3. *The principal's equilibrium payoff in any ex ante contracting environment is strong set order greater than her equilibrium payoff set in the same environment with ex post contracting.*

4. *Ex ante contracting is superior to ex post contracting by the Potential Pareto Criterion.*

Proof:

To see the first two claims, Lemma 2 shows that the agent necessarily receives $\bar{U}$ in any ex ante contracting environment. Lemma 4 shows that in ex post contracting, he also receives $\bar{U}$ when information is symmetric and Lemma 3 shows that there exist equilibria in which he receives more than $\bar{U}$ when the principal has private information.

Third claim: That the principal's equilibrium payoff with ex ante contracting is strong set order greater than her equilibrium payoff set with ex post contracting for the same asymmetric information environment follows because for any equilibrium in ex post contracting with $\dot{I}_{\lambda(z_1)}(a_{m_{\lambda(z_1)}})$ and $\dot{I}_{\lambda(z_2)}(a_{m_{\lambda(z_2)}})$, the ex ante contract $\{\dot{I}_{\lambda(z_1)}(a_{m_{\lambda(z_1)}}), \dot{I}_{\lambda(z_2)}(a_{m_{\lambda(z_2)}})\}$ is feasible. Moreover, the principal can do better provided that, at these values, $\frac{\mu_1}{prob(z_1)} \neq \frac{\mu_2}{prob(z_2)}$ in (8).

Finally, the fourth claim follows because in any symmetric information environment, the agent is equally well off with ex ante as with ex post contracting, but the principal is better off with ex ante contracting. Thus, it remains to show that there is some contract that yields the agent the same utility and the principal greater profit.

Consider the least-cost separating equilibrium from some asymmetric information, ex post contracting environment. If these contracts constituted the ex ante contract, they would provide the agent the same expected utility and the principal the same expected cost. Entering the expected utility from each ex post contract for u_k^* in (8) yields the same solution for each program. Except in the degenerate case where $\frac{\mu_k}{prob(z_k)}$ are equal for each z_k, the principal can provide the same utility to the agent at a lower cost. $\frac{\mu_k}{prob(z_k)}$ is the marginal cost of utility provision. Reducing the utility provided by one announcement-contingent contract and increasing the utility provided by another announcement-contingent contract with a smaller $\frac{\mu_k}{prob(z_k)}$ yields a smaller cost and the same utility. The principal can adjust the contract to provide the agent expected utility that exceeds his expected utility in the least-cost separating equilibrium with ex post contracting. By increasing the agent's utility some small amount, both the principal and the agent will be better off.

By adjusting the ex ante contract to lower her cost and provide the agent the same utility level, the principal may implement a different action profile in ex ante contracting. Because the same action profile is feasible at a lower cost, if she implements a different action profile, then she would do even better. ■

5 Conclusion

We have analyzed a principal-agent model with moral hazard where the environment depends on whether the principal has perfect or imperfect information about her technology, and whether she has private or symmetric information. We have obtained results that, when information is symmetric, do not arise. In addition, we have examined the value of information when the principal has private information and then contrasted ex post with ex ante contracting.

Chade and Silvers [1] examined a third-best situation that departs from the second-best in that the principal has private information.[10] In the current paper, the consequences of a further departure – imperfect information about the technology – have been explored. We've shown that the principal may lose by acquiring a more informative information structure about the technology. Although it may induce the principal to implement actions that are closer to third-best, the marginal insurance value of the more informative information structure is actually negative with ex post contracting. Thus, with ex post contracting, a more informative information structure makes the principal worse off unless she implements a non-constant action profile.

[10] They showed that even when the principal has perfect information, because it is private, she may implement a different action profile than is second-best.

In contrast, in ex ante contracting, the principal can effectively adjust the contracts to insure against the possibility of having the less informative technology; thus, a more informative information structure makes her better off. Moreover, although the agent prefers that the principal has private information with ex post contracting, both complete information ex post contracting and ex ante contracting when the principal will have private information, are superior to ex post contracting when the principal has private information, by the Potential Pareto Criterion.

The analysis extends the understanding of the agency literature with moral hazard and privately informed principal, specifically, the consequences of the principal knowing more accurately her technology. An interesting extension would be to explore the consequences of the agent having greater symmetry of information. This could help to explain whether or when firms or neutral third parties improve welfare by providing credible, public information.

The model also can naturally be extended to other types of private information, such as the agent's disutility of effort or reservation utility. This would help to determine whether the consequences to private information about the technology are due to the existence of private information or the type of private information. Additionally, it may be of interest to examine situations in which the agent has, or may come to have, private information.

6 Appendix

6.1 Equilibria and Equilibrium Contracts

A Perfect Bayesian Equilibrium (PBE) is a specification of strategies for each player and beliefs such that the strategies are sequentially rational given the beliefs and the beliefs are consistent and derived using Bayes' rule on the equilibrium path. Note that a PBE must also specify beliefs off the equilibrium path, and the players must respond rationally given those beliefs, but that the beliefs can be chosen arbitrarily.

In this context, for an environment, a PBE must specify a contract for the principal to offer conditional upon the principal's type, whether the agent accepts or rejects the contract, an action for the agent to take, and beliefs. In ex ante contracting, a PBE must also specify an announcement by the principal after the agent accepts the contract. Both the principal and the agent have a common prior, λ, that the principal has Π_1. The principal forms her posterior beliefs after receiving z_k. The agent forms interim beliefs after observing the event that contains z_k as determined by his information function; he then forms posterior beliefs after receiving the principal's contract offer or announcement in ex post or ex ante contracting. These beliefs follow Bayes' rule. Finally, beliefs for all other possible contracts in ex post contracting, and additionally for all other announcements in ex ante contracting, must be specified, though these beliefs are not restricted.

In ex post contracting, the principal offers a contract that specifies a wage for each possible outcome. In ex ante contracting, the principal offers an ex ante contract that specifies a wage contingent upon the principal's announcement, which must also be specified, and the outcome.

Note that when information is not symmetric, it is necessary for the contract choice to be incentive compatible for the principal, since neither the agent, nor any third party, can know the signal that the principal observed.

The agent's acceptance/rejection decision depends upon whether the contract is individually rational. In ex post contracting, a contract is individually rational if and only if (1) holds, where λ is the agent's posterior belief. Denote the left-hand side of (1) by $EU(I)$. In ex ante contracting, an ex ante contract is denoted by $\{I_{\lambda(z_1)}, I_{\lambda(z_2)}\}$, where $I_{\lambda(z_k)}$ is the announcement-contingent contract if the principal announces $\Pi_{\lambda(z_k)}$. An ex ante contract is individually rational if

$$prob(z_1)EU(I_{\lambda(z_1)}) + prob(z_2)EU(I_{\lambda(z_2)}) \geq \bar{U} \tag{9}$$

The agent's action choice is the argmax of his expected utility, so that he chooses a_m if and only if (2) holds. That is, an ex post contract is incentive compatible if and only if (2) holds. An ex ante contract is incentive compatible if and only if for each possible announcement, $\Pi_{\lambda(z_k)}$, (2) holds. Note that incentive compatibility depends on the agent's posterior belief, which is a function of z_k and the principal's incentives to report truthfully.

A principal of a certain type may deviate by either offering a different contract that implements the same action or that implements a different action. In each equilibrium, beliefs that the agent holds for all contracts not offered in equilibrium, such that no type gains by deviating to such a contract, must be specified. The out-of-equilibrium beliefs can be any value $\lambda \in [0, 1]$; the beliefs partition the space of feasible contracts both into those that are individually rational and those that are not, and into those that are incentive compatible and those that are not. More generally, the out-of-equilibrium beliefs can be any function mapping from the space of feasible contracts into $[0, 1]$, thereby yielding specific individual rationality and incentive compatible constraints.

6.1.1 Ex Post Contracting

For ex post contracting, as in many signaling games, both separating and pooling equilibria exist. It is worth noting as a technical point that any uncertainty has a profound effect on the equilibrium payoff correspondence. Specifically, in the *Asymmetric Imperfect Information* environment, both at $\lambda = 0$ and $\lambda = 1$, neither the principal's nor the agent's payoff correspondence is lower hemi-continuous. Additionally, if the agent's beliefs are sufficiently pessimistic, then, both at $\lambda = 1$, and at $\lambda = 0$ if also only pooling equilibria obtain, neither correspondence is upper hemi-continuous.

- Separating Equilibria

  In order to find a separating equilibrium, the principal of each type first determines the least-cost contract to implement an action, and then, having computed the expected cost and revenue for each action, chooses to implement the action that yields the maximum expected profit. The contract implements the desired action if and only if it is both individually rational and incentive compatible.

If the principal has private information, then the set of contracts that implement an action depends upon the agent's beliefs of the principal's type. We focus on situations in which there is no natural separation, so that if $\Pi_{\lambda(z_1)}$ were to offer $I^*_{\lambda(z_1)}$, then $\Pi_{\lambda(z_2)}$ would also offer it. If this is not true, then the different types naturally separate and there is no loss due to informational asymmetries, since $\Pi_{\lambda(z_1)}$ can offer $I^*_{\lambda(z_1)}$ and implement the appropriate action. Note that if the principal does not have perfect information, then this may not be the first-best action, for her posterior beliefs may be such that she implements a higher or lower action than is optimal with perfect information.

If $\Pi_{\lambda(z_2)}$ mimics $\Pi_\lambda(z_1)$, then $I^*_{\lambda(z_1)}$ would not be either individually rational or incentive compatible, since the agent's equilibrium beliefs could not equal $\lambda(z_1)$. Thus, the contracts must also satisfy no-mimic constraints. $\Pi_{\lambda(z_2)}$ does not mimic $\Pi_\lambda(z_1)$ if (5) holds.

The agent's choices are then to accept or reject these contracts and choose the appropriate actions. The two types need not implement the same action, and it is even possible for the worse type to implement a higher action.

Finally, if the principal has private information, then a separating equilibrium must specify beliefs on the equilibrium path and off the equilibrium path. Because this is a separating equilibrium, the agent's beliefs are $\lambda(z_1)$ with probability one given $I_{\lambda(z_1)}$, and $\lambda(z_2)$ with probability one given $I_{\lambda(z_2)}$. There are infinitely many separating equilibria.

- Pooling Equilibria

  For any pooling equilibrium, both types offer the same contract so that the agent's posterior beliefs equal his interim beliefs. The pooling equilibrium exists then if the contract is individually rational and incentive compatible given the agent's posterior beliefs which equal his prior beliefs. Let the agent hold beliefs that the principal is $\Pi_{\lambda(z_2)}$ for any other contract. Given this, neither type would deviate to a contract that satisfies $IR(\lambda(z_2), a_m)$ and $IC(\lambda(z_2), a_m, a_{\tilde{m}})$. Note that for $\Pi_{\lambda(z_2)}$, this would be $I^*_{\lambda(z_2)}$, but for $\Pi_{\lambda(z_1)}$ it may be some other contract. I^*_λ is a feasible pooling equilibrium contract, though there are infinitely many.

  In addition, the agent may hold beliefs such that if I^*_λ were offered, he would not believe the principal is Π_λ so that he would either reject the contract or not choose a_m.

6.1.2 Ex Ante Contracting

For ex ante contracting, the principal offers an ex ante contract that may be separating or pooling.

- *Complete Information*

  The principal can offer the ex ante contract consisting of $I^*_1(a_{m_1})$ and $I^*_0(a_{m_0})$ but may do better by trading off utility provision. Because they will have symmetric information, there is no principal's incentive compatibility constraint. Note that I^*_λ is not feasible since it does not implement a_{m_λ} when the principal has Π_0.

The optimal contract is denoted $\{I_1^{**}(a_{m_1}), I_0^{**}(a_{m_0})\}$.

- *Asymmetric Perfect Information*

  As with ex post contracting, pooling and separating contracts are both possible. Because the principal will have private information, the ex ante contract must induce the principal to truthfully report her type.

  The ex ante contract consisting of $\hat{I}_1(a_{m_1})$ and $I_0^*(a_{m_0})$ is a possible ex ante contract, but the principal may do better by trading off utility from one announcement-contingent contract for utility in another one.

  The cost of symmetric information contracts is convex in the following sense:

  Lemma 5 *Convexity in Technology of the Expected Cost of Symmetric Information Contracts*

  Let λ be the prior probability of the principal having Π_1 and, without loss of generality, let $(1-\lambda') < \lambda$. The expected cost of symmetric information contracts for $\Pi_{\lambda'}$ and $\Pi_{(1-\lambda')}$ is greater than the cost of a pooling contract, I_λ^*.

  Proof: See Appendix 6.2.

  Setting $\lambda' = 1$ in the lemma, we have

$$\lambda C_1(I_1^*(a_m)) + (1-\lambda)C_0(I_0^*(a_m)) > C_\lambda(I_\lambda^*(a_m)); \text{thus,}$$

  if the principal implements the same action given z_1 or z_2, then the principal offers I_λ^* whether she announces Π_1 or Π_0; else, she offers a least-cost separating contract denoted $\{\hat{\hat{I}}_1(a_{m_1}), I_0^{**}(a_{m_0})\}$. This maximizes

$$\lambda\Big(B(a_{m_1}) - C_1(\hat{\hat{I}}_1(a_{m_1}))\Big) + (1-\lambda)\Big(B(a_{m_0}) - C_0(I_0^{**}(a_{m_0}))\Big)$$

  where $\hat{\hat{I}}_1(a_{m_1})$ satisfies (2) for Π_1, $I_0^{**}(a_{m_0})$ satisfies (2) for Π_0, and together they satisfy (9) and $PIC_{\lambda(z_1)\lambda(z_2)}$.

- *Asymmetric Imperfect Information*

  This is similar to *Asymmetric Perfect Information* except that instead of the principal being Π_1 or Π_0, she is either $\Pi_{\lambda(z_1)}$ or $\Pi_{\lambda(z_2)}$. We assume, without loss of generality, that $\zeta_{11} > \zeta_{01}$, so that $\lambda(z_1) > \lambda > \lambda(z_2)$.

6.2 Proofs

Proof of Lemma 3:

Consider I^*_λ, the complete information ex post contract offered by Π_λ. This ex post contract cannot be offered because, by assumption, it does not satisfy the constraint (5). Π_λ then alters the ex post contract at the smallest increase in cost while still satisfying individual rationality (1) and incentive compatibility (2) but increasing the cost to $\Pi_{\lambda'}$ enough to dissuade $\Pi_{\lambda'}$ from mimicking. We show that altering the ex post contract while holding expected utility at $\bar{U}$ cannot be optimal. Thus, since individual rationality must still be satisfied, it is satisfied with strict inequality.

Given the objective of increasing cost on $\Pi_{\lambda'}$ as much as possible for each unit increase in cost of Π_λ, the gradient of this relative cost increase is

$$\Big(\frac{\pi_{\lambda'1}(a_m)}{\pi_{\lambda 1}(a_m)}, \frac{\pi_{\lambda'2}(a_m)}{\pi_{\lambda 2}(a_m)}, \ldots, \frac{\pi_{\lambda' N}(a_m)}{\pi_{\lambda N}(a_m)}\Big).$$

Let $IC(\lambda, a_m, a_{\tilde{m}})$ be a binding incentive compatibility constraint at I^*_λ. Suppose $q_n > q_{\tilde{n}}$ and consider altering I^*_λ by increasing $I_{\lambda n}$ by one and changing $I_{\lambda\tilde{n}}$ by $-\frac{\pi_{\lambda n}(a_m)-\pi_{\lambda n}(a_{\tilde{m}})}{\pi_{\lambda\tilde{n}}(a_m)-\pi_{\lambda\tilde{n}}(a_{\tilde{m}})}\frac{V'(I_n)}{V'(I_{\tilde{n}})}$. In $\Re^N$, this change can be described by a vector in which each element equals 0 except the n and $\tilde{n}$ elements that equal one and the value above, respectively. This vector is a directional vector along the incentive compatibility constraint.

Similarly, a directional vector that holds the agent's expected utility constant for selecting a_m and increases $I_{\lambda n}$ by one, changes $I_{\lambda\tilde{n}}$ by $-\frac{\pi_{\lambda n}(a_m)}{\pi_{\lambda\tilde{n}}(a_{\tilde{m}})}\frac{V'(I_n)}{V'(I_{\tilde{n}})}$. Each element of this vector in $\Re^N$ in this direction equals 0 except the n and $\tilde{n}$ elements that equal one and the value above, respectively.

Denoting these directional vectors by r_{IC} and r_{IR}, respectively, the directional derivatives along an incentive compatibility constraint and along an agent's indifference curve are given by:

$$\Big(-\frac{\pi_{\lambda'\tilde{n}(a_m)}}{\pi_{\lambda\tilde{n}(a_m)}}\frac{\pi_{\lambda n}(a_m)-\pi_{\lambda n}(a_{\tilde{m}})}{\pi_{\lambda\tilde{n}}(a_m)-\pi_{\lambda\tilde{n}}(a_{\tilde{m}})}\frac{V'(I_n)}{V'(I_{\tilde{n}})} + \frac{\pi_{\lambda' n(a_m)}}{\pi_{\lambda n(a_m)}}\Big)/||r_{IC}||$$

and

$$\Big(-\frac{\pi_{\lambda'\tilde{n}}(a_m)}{\pi_{\lambda\tilde{n}}(a_m)}\frac{\pi_{\lambda n}(a_m)}{\pi_{\lambda\tilde{n}}(a_{\tilde{m}})}\frac{V'(I_n)}{V'(I_{\tilde{n}})} + \frac{\pi_{\lambda' n(a_m)}}{\pi_{\lambda n(a_m)}}\Big)/||r_{IR}|| \ .$$

Because the actions generate different distributions over the outcomes, there exist n and $\tilde{n}$ such that MLRP holds with strict inequality. Consider the two differences $\pi_{\lambda\tilde{n}}(a_m) - \pi_{\lambda\tilde{n}}(a_{\tilde{m}})$ and $\pi_{\lambda n}(a_m) - \pi_{\lambda n}(a_{\tilde{m}})$. There are four cases.

- the first difference is negative and the second difference is positive. But then, the directional derivative along the individual rationality constraint, which is the agent's indifference curve corresponding to the utility level $\bar{U}$, is strictly less than that along the incentive compatibility constraint. To see this, the inequalities imply that the first term in the numerator of the directional derivative along the incentive compatibility constraint is positive while that of the directional derivative along the individual rationality constraint is negative. The claim follows since MLRP states $\frac{\pi_{\lambda n}(a_m)}{\pi_{\lambda \tilde{n}}(a_m)} > \frac{\pi_{\lambda n}(a_{\tilde{m}})}{\pi_{\lambda \tilde{n}}(a_{\tilde{m}})}$ which implies that $\frac{\pi_{\lambda n}(a_m)}{\pi_{\lambda \tilde{n}}(a_m)} > \frac{\pi_{\lambda n}(a_m)-\pi_{\lambda n}(a_{\tilde{m}})}{\pi_{\lambda \tilde{n}}(a_m)-\pi_{\lambda \tilde{n}}(a_{\tilde{m}})}$, which means that $||r_{IC}|| < ||r_{IR}||$.

- both differences are negative. But this implies that for incentive compatibility to remain satisfied with equality, an increase in $I_{\lambda n}$ implies a decrease in $I_{\lambda \tilde{n}}$ that is smaller than the decrease implied by holding utility constant at $\bar{U}$. Therefore, since the first difference is negative and $I_{\lambda \tilde{n}}$ is diminished less than is necessary, incentive compatibility holds with strict inequality.

- both differences are positive. But this implies that for incentive compatibility to remain satisfied with equality, an increase in $I_{\lambda n}$ implies a decrease in $I_{\lambda \tilde{n}}$ that is larger than the decrease implied by holding expected utility constant at $\bar{U}$. Therefore, since the first difference is positive, altering the ex post contract to maintain expected utility at $\bar{U}$ implies that incentive compatibility holds with strict inequality.

- the first difference is positive and the second difference is negative. However, this case is impossible since it violates MLRP as these two inequalities imply $\pi_{\lambda n}(a_{\tilde{m}}) > \pi_{\lambda n}(a_m)$ and $\pi_{\lambda \tilde{n}}(a_{\tilde{m}}) < \pi_{\lambda \tilde{n}}(a_m)$, which together yields $\frac{\pi_{\lambda n}(a_{\tilde{m}})}{\pi_{\lambda \tilde{n}}(a_{\tilde{m}})} > \frac{\pi_{\lambda n}(a_m)}{\pi_{\lambda \tilde{n}}(a_m)}$.

If either the second or third case above holds, then every incentive compatibility constraint is now relaxed. Consider the following modification to an ex post contract that satisfies individual rationality with equality:

Let $\frac{\pi_{\lambda n}(a_m)}{\pi_{\lambda \tilde{n}}(a_m)} > \frac{\pi_{\lambda' n}(a_m)}{\pi_{\lambda' \tilde{n}}(a_m)}$.[11]

MLRP implies that $\frac{\pi_{\lambda n}(a_m)}{\pi_{\lambda \tilde{n}}(a_m)} > \frac{\pi_{\lambda n}(a_{\tilde{m}})}{\pi_{\lambda \tilde{n}}(a_{\tilde{m}})}$.

Decreasing $I_{\lambda n}$ by some $\epsilon > 0$ and increasing $I_{\lambda \tilde{n}}$ by $\epsilon \frac{\pi_{\lambda n}(a_m)}{\pi_{\lambda \tilde{n}}(a_m)}$ yields an ex post contract that:

- costs Π_λ the same as the proposed ex post contract does;

- costs $\Pi_{\lambda'}$ strictly more than the proposed ex post contract does; and

- yields the agent more utility than the proposed ex post contract does.

This alteration to the ex post contract is feasible since the incentive compatibility constraints were not binding. Π_λ could then decrease $I_{\lambda \tilde{n}}$ without violating any constraint and implement a_m at a lower cost.

Any changes to several I_n can be expressed as the weighted sum of several binary changes. The only way in which the agent would receive $\bar{U}$ from a feasible ex post contract would be if the directional derivative along individual rationality exceeded that along any incentive compatibility constraint. But then, the incentive compatibility constraints would all be relaxed so that the principal could do even better by altering the ex post contract as above.

[11] If this inequality is reversed, then the analysis proceeds by increasing $I_{\lambda n}$ and decreasing $I_{\lambda \tilde{n}}$, in what follows.

Because any movements along the incentive compatibility constraints that still satisfy individual rationality must satisfy the individual rationality constraint with slack, the agent receives more than $\bar{U}$ in the least-cost separating ex post contract offered by Π_λ. ■

Proof of Lemma 5:

Let $p_{\lambda'} = \frac{\lambda+\lambda'-1}{2\lambda'-1}$.[12] Note that $\sum_{n=1}^{N} \pi_{\lambda' n}(a_m)V(I^*_{\lambda' n}) - a_m = \bar{U}$ and $\sum_{n=1}^{N} \pi_{(1-\lambda')n}(a_m)V(I^*_{(1-\lambda')n}) - a_m = \bar{U}$, so that

$$p_{\lambda'}\Big(\sum_{n=1}^{N}\pi_{\lambda' n}(a_m)V(I^*_{\lambda' n}) - a_m\Big) + (1-p_{\lambda'})\Big(\sum_{n=1}^{N}\pi_{(1-\lambda')n}(a_m)V(I^*_{(1-\lambda')n}) - a_m\Big) =$$
$$\sum_{n=1}^{N} p_{\lambda'}\pi_{\lambda' n}(a_m)V(I^*_{\lambda' n}) - p_{\lambda'}a_m + \sum_{n=1}^{N}(1-p_{\lambda'})\pi_{(1-\lambda')n}(a_m)V(I^*_{(1-\lambda')n}) - (1-p_{\lambda'})a_m = \bar{U}$$

In addition, $\forall \tilde{m} \neq m \quad \sum_{n=1}^{N}(\pi_{\lambda' n}(a_m) - \pi_{\lambda' n}(a_{\tilde{m}}))V(I^*_{\lambda' n}) \geq a_m - a_{\tilde{m}}$ and $\sum_{n=1}^{N}(\pi_{(1-\lambda')n}(a_m) - \pi_{(1-\lambda')n}(a_{\tilde{m}}))V(I^*_{(1-\lambda')n}) \geq a_m - a_{\tilde{m}}$, so that

$$p_{\lambda'}\Big(\sum_{n=1}^{N}(\pi_{\lambda' n}(a_m) - \pi_{\lambda' n}(a_{\tilde{m}}))V(I^*_{\lambda' n})\Big) +$$
$$(1-p_{\lambda'})\Big(\sum_{n=1}^{N}(\pi_{(1-\lambda')n}(a_m) - \pi_{(1-\lambda')n}(a_{\tilde{m}}))V(I^*_{(1-\lambda')n})\Big) \geq$$
$$p_{\lambda'}(a_m - a_{\tilde{m}}) + (1-p_{\lambda'})(a_m - a_{\tilde{m}}) = a_m - a_{\tilde{m}}$$

Thus, an ex post contract that gives the agent $I^*_{\lambda' n}$ with probability $p_{\lambda'}\pi_{\lambda' n}(a_m)$ and $I^*_{(1-\lambda')n}$ with probability $(1-p_{\lambda'})\pi_{(1-\lambda')n}(a_m)$, when summed over all possible outcomes, would give the agent exactly his reservation utility and would implement a_m. But, I^*_λ is the least-cost contract among those that satisfy individual rationality and incentive compatibility for Π_λ. ■

[12] For these values, $\lambda' > \lambda$ so that $p_{\lambda'} \in (0,1)$ and $p_{\lambda'}\pi_{\lambda' n}(a_m) + (1-p_{\lambda'})\pi_{(1-\lambda')n}(a_m) = \pi_{\lambda n}(a_m)$.

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