

Market Thickness, Labor Market Flexibility and Wage Dynamics*

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Abstract

We study the link between market thickness, labor market flexibility and wage dynamics. We consider an economy with two sectors; a risk-free sector that employs workers only, and a risky sector with matching frictions that employs both workers and employers. Workers are risk-averse, whereas employers are risk-neutral. In the risky sector, complete contracts are unavailable due to informational reasons; hence flexible self-enforcing contracts are the only means to share risk. We show that shifts out of stable employment into flexible employment engendered by improvements in search effectiveness increases the average real wage and wage volatility in the risky sector while raising the (expected) real wages and worker welfare in the whole economy. Further, depending on parameter values, it may also increase economy-wide real wage volatility. Therefore, our model can explain the transitory variation in workers' earnings observed during 1970s and 1980s, even for job stayers.

Keywords: Invisible Handshake; Incomplete Contracts; Wage Volatility; Search Frictions.

JEL classification: E24, J31, J63, J64.

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1 Introduction

“People need to look at themselves as self-employed, as vendors who come to this company to sell their skills. In AT&T, we have to promote the whole concept of work force being contingent, though most of the contingent workers are inside our walls. Jobs are being replaced by projects and fields of work, giving rise to a society that is increasingly jobless but not workless.” – James Meadows, AT&T Vice President for Human Resources [excerpt taken from the article written by Edmund L. Andrews in New York Times, on February 13th, 1996].

In the second half of the twentieth century, with recent advances made in information technology, the world has become much more interconnected than before. The rapid diffusion of information accompanied by ever decreasing communication costs accelerated the process of globalization.¹ This greater openness has brought many benefits, yet it may have important social costs. One such cost that is put forth in popular accounts is the rising insecurity and risk for workers.² This is what we explore in this paper.

During the same time period when globalization has gained momentum, the U.S. labor market has undergone some discernible changes. First, it has become much more flexible. Prior to 1970s, the long-term steady employment relationship was the norm. The stable, ordinary assembly jobs with high degrees of job security dominated the economy. However, a major turn of events took place in the early 1970s with the decline of routine mass-production jobs. The nature of employment relationship was altered to accommodate high levels of turnover, shorter periods of employment and the profound use of contingent employment contracts. The growth of temporary help services industry in the U.S. documented in the literature³ is one example of such flexible employment arrangements. The upshot is that the U.S. labor market has experienced greater instability⁴ and weaker ties between workers and

¹See Korzeniewicz and Gereffi (1994), Castells (1996) and Sassen (1996).

²See, for example, Gosselin (2008) and Hacker and Jacobs (2008).

³Temporary help services industry consists of agencies that find workers for client firms to do temporary jobs. Since 1972, employment in temporary help services has grown at 11 percent per year. More on this can be found, among others, in Segal and Sullivan (1997) and Autor, Levy and Murnane (1999).

⁴See Farber (2009).

employers — greater flexibility.^{5,6}

Second, as a result of advancements in information technology, the functioning of factor markets has become much more efficient than before. This is manifested by the rising role played by labor market intermediaries in response to drastic changes observed in employment conditions.⁷ The increase in the number of intermediaries such as temporary help firms, headhunters,⁸ and internet-based job sites and social networks can in fact raise the probability of finding a job for a worker or of filling a vacancy for an employer in a given time period without any change in the number of participants or their attributes. Therefore, the improvements in search (matching) efficiency can make labor markets thicker.⁹ Katz and Krueger (1999) and Autor (2001a) provide evidence that the surge of temporary help agencies since 1970s increased matching efficiency. A similar efficiency in search can also be due to the emergence of headhunters as documented by Finlay and Coverdill (2002). They state that headhunters' knowledge of labor market allows them to connect employers with workers, as a result generating matches that would not have occurred otherwise. This efficiency effect is more pronounced especially after 1970s when the headhunting industry experienced major changes.¹⁰ In addition, the advent of internet was a turning point for the way labor market operates. The arrival of internet websites such as Monster.com, LinkedIn and Facebook has remarkably altered how employers and workers search for each other, see Autor (2001b) for examples. The internet-based job sites and social networks have many advantages over the conventional methods of job search. They are easier to use and the information provided

⁵See Benner (2002).

⁶This observed trend is not particular to the U.S. In G-7 countries, 30 to 45 percent of all workers have some form of flexible employment and this ratio is increasing. See Carnoy and Castells (1997).

⁷Labor market intermediaries are those institutions that mediate work practices and provide matching activities for employers and workers. See Kazis (1998) for the extended role played by labor market intermediaries.

⁸Headhunters are third-party agents who find job candidates for employers for a fee.

⁹McLaren (2003) identifies three different ways that market thickness can occur: (a) rise in the number of market participants, (b) increased versatility of participants, and (c) improvements in search efficiency. Our focus in this paper is on the last one.

¹⁰Finlay and Coverdill (2002) identify two major changes. First, the payment of fees for headhunters shifted from job seekers to employers. Second, headhunters started to generate candidates for positions. This is quite different than what traditional employment agencies do, which is finding jobs for people, rather than finding people for jobs.

is less costly and more up-to-date. As a result, these improvements in search technology suggest a significant reduction in matching frictions. This claim finds support in data, see for example Stevenson (2006), Kuhn (forthcoming) and Kuhn and Mansour (forthcoming).¹¹

Third, there has been a rise in the volatility of wages (short-term earnings variance) in the United States over 1970s and 1980s as documented by Gottschalk, Moffitt, Katz and Dickens (1994) and Kambourov and Manovskii (2009).¹² The analysis of Gottschalk, Moffitt, Katz and Dickens (1994) yield that much of the increase in the earnings volatility in the 1980s has arisen within jobs, and earnings instability has also increased even for job stayers. On the other hand, Kambourov and Manovskii (2009) show that the increase in short-term variability of earnings is consistent with the rise in occupational mobility.

In this paper, we ask whether there is any connection between these developments. In other words, whether the rise in market thickness brought by improvements in search effectiveness can cause an increase in short-term earnings variance by increasing labor market flexibility.¹³ We have a stylized model that connects these pieces together. We consider a risk-bearing employment relationship between risk-averse workers and risk-neutral employers in a labor market with search frictions. The environment is risky and complete contracts are unavailable due to informational reasons. Thus, the only way for an employer to share risk with a worker is to develop long-term employment agreement, in which employers promise wage insurance while purchasing labor. These implicit contracts,¹⁴ also known as ‘invisible handshake’, are enforceable only through the threat that if any agent reneges, the relationship is severely damaged such that it is dissolved and parties to the contract must search for

¹¹There are other studies that find little or no such evidence of internet’s effect on unemployment duration or unemployment rate, see Kuhn and Skuterud (2004) for the former and Kroft and Pope (forthcoming) for the latter. On the other hand, Kuhn and Mansour (forthcoming) replicate the study by Kuhn and Skuterud (2004) with new survey data and find that the earlier (negative) results are reversed. In addition, Stevenson (2006) shows that the vast majority of online job-seekers are those who are already employed and that employer-to-employer worker flows rise with the use of internet.

¹²A rise in earnings volatility during the same period is also documented for Canada (Baker and Solon, 2003) and Great Britain (Dickens, 2000).

¹³Using a search model of labor market with two-sided heterogeneity and risk-neutral agents, Uren (2008) shows that reduced search frictions increase wage inequality by increasing the degree of assortive matching. His focus is different than ours, however, since he does not address issues around risk.

¹⁴See Beaudry and DiNardo (1991) and McDonald and Worswick (1999) for empirical relevance of implicit contracts. Malcomson (1999, section 3) provides a survey.

new partners. An increase in market thickness due to an improvement in search technology makes it easier to find a new partner (either worker or employer) to work with and makes the termination of a given relationship less intimidating. This in turn weakens risk-sharing, makes existing relationships less stable and increases the volatility of wages.

There are other papers in the literature that analyze the relationship between market thickness and self-enforcing relationships. Kranton (1996) identifies that larger markets can destroy long-run relationships whereas increase in market search costs can facilitate them. Ramey and Watson (2001) study the effect of matching frictions on investment incentives of agents in a bilateral self-enforcing trading relationships. By confining their attention to stationary risk-sharing relationships, McLaren and Newman (2004) show that reductions in market frictions can potentially weaken cooperation and reduce welfare by increasing agents' outside options. When there is information asymmetry, Matouschek and Ramezzana (2007) show that an improvement in search frictions can make bilateral exchange more difficult. However, none of these papers are particular to labor market and wage volatility. Instead, our paper is closely related to Thomas and Worrall (1988) and Karabay and McLaren (2010). Thomas and Worrall examine long-run relationships between a risk-averse worker and a risk-neutral employer when each can alternatively participate an exogenous and randomly fluctuating labor spot market. The wage agreement within a given relationship is generally tethered by the ongoing wage in the spot market. Karabay and McLaren (2010) extend the work of Thomas and Worrall (1988) by endogenizing the spot market and adding moral hazard. They analyze the effects of free trade and offshoring on wage volatility and worker welfare. *In this paper, we add an important element to Karabay and McLaren (2010), namely, search effectiveness and show that, besides other factors such as free trade (through price effect) and offshoring (through integration effect), improvements in search effectiveness can also affect wage volatility and worker welfare. In our analysis, we look at not only the wage volatility in a particular sector as in Karabay and McLaren (2010) but also economy-wide wage volatility.*

In our stylized model, we have two sectors, a 'careers sector' in which production is

risky and requires unobservable effort by a worker and by an employer, and a ‘spot market sector’ with risk-free Ricardian technology. Here, the ‘careers sector’ represents a sector that is heavily characterized by flexible employment relationship such as services, and the ‘spot market sector’ represents a sector like manufacturing that involves routine production jobs.¹⁵ We have three essential elements that shape our approach. The first element is the existence of non-diversifiable firm-specific risk. In the ‘careers sector’, firms are assumed to be hit by idiosyncratic shocks that are unobservable to people outside the firm. As a result, written contracts are not enforceable. Second, in the ‘careers sector’, there is no commitment. Since employers are risk-neutral, whereas workers are risk-averse, employers would like to commit credibly to a full wage insurance (constant wage), in effect reducing their expected wage payments to workers; but without enforceable contracts, they can only do so by reputational means (self-enforcing contracts). Consequently, in providing wage insurance, employers are constrained by their incentive compatibility constraints. The last element to our model is moral hazard. For production to occur in the ‘careers sector’, workers need to exert effort that is costly, unobservable and thus non-contractable. Therefore, in equilibrium, the wage payments are back-loaded in this sector.¹⁶ The intuition is that when a worker needs to provide a non-contractable effort, it is generally optimal to promise wages that increase over time, so that the fear of losing high future wages deters shirking. Thus in the risky sector, new workers are always cheaper than incumbent workers. This is the essence of the employer’s problem: if it is easy to find a replacement, the employer has a temptation to ditch the current senior worker for a new cheaper worker and this temptation is strongest if the firm is in difficulty. When this is the case, workers will know not to trust the employer’s full wage insurance, and expecting a low wage in bad times, they will demand a high wage in good times. Therefore, if it is easy to find a new worker, an employer that makes only credible promises will promise a low wage in bad states and a high wage in good states,

¹⁵Therefore, any move from spot market sector to careers sector (i.e., from manufacturing to services) represents a shift from more stable jobs to less stable ones. Schettkat and Yocarini (2006) reviews the literature analyzing the shift to services.

¹⁶This result is in the same spirit as Lazear (1979), Harris and Holmström (1982), Holmström (1983) and Shleifer and Summers (1988).

causing wage volatility in equilibrium. An increase in market thickness brought about by an advancement in search technology makes it easier for firms to hire workers, which in turn reduces the amount of wage insurance promised, raising the variance of wages in the ‘careers sector’.

The rest of the paper is organized as follows. The next section lays out our model. In section 3, we characterize optimal wage contracts. Sections 4 and 5 derive the conditions under which those contracts will exhibit constant wages and volatile wages, respectively. Comparative statics of wage dynamics is analyzed in section 6. In section 7, we turn to general equilibrium analysis. The last section concludes. All proofs are relegated to the appendix.

2 The Model

In this section, we describe the main features of our model. The setup is based on Karabay and McLaren (2010). That paper analyzes the effect of trade and offshoring on wage volatility, and therefore considers two-good, two factor and two-country model. In contrast, our focus in this paper is the effect of an improvement in search technology on wage dynamics and therefore, we consider two-good, two factor model within a single country. First, we review the main model characteristics and then analyze how they change as labor market becomes thicker through an improvement in search effectiveness.

Production. There are two sectors; Y and X , and two factors of production: a measure L of workers and a measure E of employers. In the risk-free sector (spot-market sector), Y , there is a linear production technology such that one unit of worker can produce one unit of output per period. Let $p^y > 0$ represent the price of Y -sector output. Since we have constant returns to scale technology with only one factor, the wage workers earn in this sector must be $\omega^y = p^y$.

In the risky sector (careers sector), X , for production to occur one worker must team up with one employer and they must each put in one unit of non-contractible effort. We

will call a given such partnership as a ‘firm’. Workers suffer a disutility from effort equal to $k > 0$, while employers suffer no such disutility.¹⁷ Within a given employment relationship, denote the effort put in by agent i by $e^i \in \{0, 1\}$, where $i = W$ indicates the worker and $i = E$ indicates the employer. Let sector X be the numeraire sector, i.e., $p^x \equiv 1$. The output and revenue generated in that period is then equal to $x_\epsilon e^W e^E$, where, ϵ is an idiosyncratic i.i.d. random variable that takes its value $\epsilon = G$ or B with respective probabilities π_G and π_B , where $\pi_G + \pi_B = 1$ and $x_G > x_B > 0$. The random variable ϵ indicates whether the current period is one with a good state or a bad state for the firm’s profitability. The average revenue is denoted by $\bar{x} \equiv \pi_G x_G + \pi_B x_B$. Employers without a worker are ‘with vacancy’ and do not produce anything.

Preferences. There is no storage, saving or borrowing. An agent’s income in a given period is equal to that agent’s consumption in that period.

Employers. Employers have the same linearly homogeneous and quasi-concave per-period utility function $U(c^X, c^Y)$, defined over consumption of goods X and Y , (c^X and c^Y), respectively. Since employers are risk-neutral, their indirect utility is a linear function of income and is given by $v(p^x, p^y, I) = \frac{I}{\Gamma(p^x, p^y)}$, where I is income and $\Gamma(p^x, p^y)$ is a linear homogeneous consumer price index that represents the minimum expenditure required to obtain unit utility. Given that $p^x \equiv 1$, $\Gamma(1, p^y)$ can be written as $P(p^y)$. Notice that the elasticity of $P(p^y)$ with respect to p^y , $\frac{p^y P'}{P}$, represents the share of good Y in total consumption, α^y , due to Roy’s identity and therefore we have $0 < \alpha^y = \frac{p^y P'}{P} < 1$.

Workers. Workers have the same per-period utility function $\mu(U(c^X, c^Y))$ over consumption of goods X and Y . The function μ is an increasing, differentiable and strictly concave von-Neumann-Morgenstern utility function. Worker’s indirect utility is given by $\mu(\frac{I}{P(p^y)})$, where the properties of the function μ guarantees that workers are risk-averse.

In short, risk-neutral employers maximize their expected discounted lifetime profits, whereas risk-averse workers maximize their expected discounted lifetime utility and everyone

¹⁷Adding a disutility for employers would not change the results other than contracting the portion of the parameter space where it is possible to have efficient and self-enforcing contracts.

discounts the future at a constant rate $\beta \in (0, 1)$.

Search. Those workers and employers seeking a partner, search until they have one. Search follows a specification of a type used extensively by Pissarides (2000). Let there be a measure n of workers and a measure m of employers searching in a given period, then $\Phi(n, m, \phi)$ matches occur. The function Φ is concave and increasing in all arguments and linear homogeneous in its first two arguments with $\Phi_{nm} = \Phi_{mn} > 0, \forall n, m, \phi$ and has an upper bound equal to $\min(n, m)$. The parameter ϕ is a measure of the effectiveness of search technology. We denote by Q^E the steady-state probability that an employer will match with a worker in any given period, or in other words, $Q^E = \frac{\Phi(n, m, \phi)}{m}$, where n and m are set at their steady-state values. Similarly, we denote by $Q^W = \frac{\Phi(n, m, \phi)}{n}$ the steady-state probability that a worker will find a job in the X sector any given period. Search has no direct cost, but it does have an opportunity cost: if an agent is searching for a new partner, then she is unable to put in effort for production with her existing partner if she has one.

There is also a possibility in each period that a worker and employer who have been matched in that period or in the past will be exogenously separated from each other. This probability is given by a constant $(1 - \rho) \in (0, 1)$.

Goods market clearing. Total production of each good must be equal to the total consumption of each good. For a given relative price p^y , each worker and employer will consume each good in the same proportions, which corresponds to the condition that $p^y = \frac{U_2(1, r)}{U_1(1, r)}$, where the subscripts denote partial derivatives and r denotes the ratio of Y production to X production. In other words, the relative price must be equal to the marginal rate of substitution between the two goods determined by the production ratio. Given that U is quasi-concave, the marginal rate of substitution is strictly decreasing in r , which in turn implies that p^y is strictly decreasing in r . We assume that $U_2(1, r) \rightarrow \infty$ as $r \rightarrow 0$, and $U_1(1, r) \rightarrow \infty$ as $r \rightarrow \infty$ so that for any $r \in (0, \infty)$, there is a unique, market clearing value of $p^y \in (0, \infty)$.

Timing of the game. The timing of events within a given period is as follows.

1. Any readily matched employer and worker learn whether they will be exogenously separated this period.
2. For each firm in the X sector, the idiosyncratic output shock ϵ is realized. This is common knowledge within the firm but unknown to agents outside the firm.
3. The wage, if any, is paid (a claim on the firm's output at the end of the period).
4. The employer and worker simultaneously choose their effort levels e^i . At the same time, the search mechanism operates. Within a firm, if $e^i = 0$, then agent i can participate in search and exert no effort. Workers in the Y sector and employers with vacancy always search and they do not incur any search cost.
5. Each firm's revenue, $R = x_e e^W e^E$, as well as profits and consumption are realized.¹⁸
6. For those agents who have found a new potential partner in this period's search, new partnerships with a new self-enforcing agreement are formed.¹⁹ This is achieved by a take-it-or-leave-it offer made by the employer to the worker. Therefore, we assume that employers have the full bargaining power.²⁰

Our focus will be on steady-state equilibria. Let V^{ES} denote the expected lifetime discounted profit of an employer with vacancy and V^{WS} denote the expected lifetime discounted utility of a searching worker without an employer (i.e., Y -sector worker), where the superscript ' S ' indicates the state of searching. Similarly, let V^{ER} and V^{WR} denote the lifetime payoffs to employers and workers, respectively evaluated at the beginning of a cooperative X -sector relationship. Naturally, we must have $V^{WR} \geq V^{WS}$ in equilibrium, or no worker will accept an X -sector job. The values V^{ij} are endogenously determined as they depend

¹⁸There is the possibility, off of the equilibrium path, that the firm's output will be zero because either agent has shirked. In such a case, we assume that the employer has deep pockets so that the wage claim promised to the worker can still be redeemed.

¹⁹Notice that for any agents matched in period t , the self-enforcing agreement is in effect in period $t + 1$ after they realized there has been no exogenous separation but before the output shock is revealed.

²⁰Assigning full bargaining power to employers simplifies the model. Our results carry over even if we allow workers to capture some portion of X -sector rents. We will comment more on this in footnote 24.

on the endogenous probability of finding a match in any given period and the endogenous value of entering a relationship once a match occurs. When designing the wage contract, any employer will take them as given. We can write

$$V^{WS} = \mu\left(\frac{\omega^y}{P}\right) + Q^W \rho \beta V^{WR} + Q^W (1 - \rho) \beta V^{WS} + (1 - Q^W) \beta V^{WS} \quad (1)$$

$$V^{ES} = Q^E \rho \beta V^{ER} + Q^E (1 - \rho) \beta V^{ES} + (1 - Q^E) \beta V^{ES} \quad (2)$$

Y -sector worker's payoff from search is the current Y -sector real wage plus the continuation values if the worker finds an X -sector job and is not immediately separated, finds an X -sector job but immediately separated, or fails to find an X -sector job. The payoff from search for an X -sector employer with vacancy is given by the continuation values if the employer finds a worker and is not immediately separated, finds a worker but immediately separated, or fails to find a worker. If an X -sector worker, or an X -sector employer who already has a worker, chooses to search, the payoff will be the same as in equations (1) and (2), respectively except for a straightforward change in the first-period payoff.

A self-enforcing agreement between a worker and an employer is simply a subgame perfect equilibrium of the game that they play together. Given that the employer has all of the bargaining power, the optimal agreement is the one that gives the highest expected discounted profit to the employer, subject to incentive constraints. If either agent reneges on the agreement, the relationship is severed and both agents must search for new partners. In other words, we will restrict our attention to 'grim punishment' strategies. Thus, the payoff following a deviation would be V^{ES} for an employer and V^{WS} for a worker.

To sum up, risk-neutral employers search for risk-averse workers, and when they find each other, the employer offers the worker the profit-maximizing self-enforcing wage contract, which then remains in force until either party reneges or the two are exogenously separated. This pattern provides a steady flow of workers and employers into the search pool, where they receive endogenous payoffs V^{WS} and V^{ES} . These values then act as parameters that constrain the optimal wage contract.

We now turn to the form of optimal contracts in the X sector.

In our model the optimal employment contracts take one of two very simple forms, which we will call ‘wage smoothing’ and ‘fluctuating wage.’ This is what we will derive in this section.

The equilibrium can be characterized as the solution to a recursive optimization problem. Let $\Omega(W)$ be the highest possible expected present discounted profit the employer can receive in a subgame-perfect equilibrium, conditional on the worker receiving an expected present discounted payoff of at least W . Arguments parallel to those in Lemma 1 of Thomas and Worrall (1988) can be used to show that the function Ω is defined on a compact interval $[W_{\min}, W_{\max}]$, decreasing, strictly concave and continuously differentiable, where W_{\min} and W_{\max} are respectively the lowest and highest worker payoffs consistent with a subgame-perfect equilibrium of the game. This function must satisfy the following functional equation

$$\Omega(W) = \max_{\{\omega_\epsilon, \widetilde{W}_\epsilon\}} \sum_{\epsilon=G,B} \pi_\epsilon \left[x_\epsilon - \omega_\epsilon + \rho\beta\Omega(\widetilde{W}_\epsilon) + (1-\rho)\beta V^{ES} \right] \quad (3)$$

subject to

$$x_\epsilon - \omega_\epsilon + \rho\beta\Omega(\widetilde{W}_\epsilon) + (1-\rho)\beta V^{ES} \geq V^{ES} \quad (4)$$

$$\mu\left(\frac{\omega_\epsilon}{P}\right) - k + \rho\beta\widetilde{W}_\epsilon + (1-\rho)\beta V^{WS} \geq V^{WS} - \mu\left(\frac{\omega^y}{P}\right) + \mu\left(\frac{\omega_\epsilon}{P}\right) \quad (5)$$

$$\sum_{\epsilon=G,B} \pi_\epsilon \left[\mu\left(\frac{\omega_\epsilon}{P}\right) - k + \rho\beta\widetilde{W}_\epsilon + (1-\rho)\beta V^{WS} \right] \geq W \quad (6)$$

$$\omega_\epsilon \geq 0 \quad (7)$$

$$\widetilde{W}_\epsilon \geq 0 \quad (8)$$

The employer’s problem stated in equation (3) is to choose the worker’s current period wage ω_ϵ and continuation utility \widetilde{W}_ϵ at each state such that the employer’s expected present discounted lifetime profit is maximized given that the worker’s expected present discounted utility is at least equal to W . Constraint (4) is the employer’s incentive compatibility constraint. If this is not satisfied in state ϵ , then the employer will in that state prefer to renege on the promised wage, understanding that this will cause the worker to lose faith in the relationship and sending both parties into the search pool. Constraint (5) is the worker’s incentive compatibility constraint. The left-hand side is the worker’s payoff from putting

in effort in the current period, collecting the wage, and continuing the relationship.²¹ The right-hand side is the payoff from shirking and searching, in which case the worker's payoff is the same as it would be if she were in the Y sector except that in the current period her income is ω_ϵ instead of ω^y . If this constraint is not satisfied, the worker will prefer to shirk by searching instead of working. Constraint (6) is the target-utility constraint. In the first period of an employment relationship, since the employer has all the bargaining power, she must promise at least as much of a payoff to the worker as remaining in the search pool would provide. Thus, in that case, denoting the target utility at the beginning of the relationship by W_0 , we have $W = W_0 = V^{WS}$ (and so $V^{ER} = \Omega(V^{WS})$). Thereafter, the employer will in general be bound by promises of payoffs she had made to the worker in the past. Finally, constraints(7) and (8) are natural bounds on the choice variables.

Constraint (5) can be replaced by the more convenient form

$$\widetilde{W}_\epsilon \geq \widetilde{W}^*, \text{ where } \widetilde{W}^* \equiv \frac{[1 - (1 - \rho)\beta] V^{WS} - \mu(\frac{\omega^y}{P}) + k}{\rho\beta}. \quad (5)'$$

The value \widetilde{W}^* is the minimum future utility stream that must be promised to the worker in order to convince the worker to incur effort and forgo search.

The following lemma allows us to ignore constraint (8) in the employer's maximization problem.

Lemma 1. $\widetilde{W}_\epsilon \geq \widetilde{W}^* > V^{WS} > 0$.

Proof. See appendix.

Given that $W_0 = V^{WS}$ and $\widetilde{W}^* > V^{WS}$, we have $W_{\min} = V^{WS}$ in the first period. Let the Kuhn-Tucker multiplier for (4) be denoted by ψ_ϵ , the multiplier for constraint (5)' by ν_ϵ , the multiplier for constraint (6) by λ , and the multiplier for constraint (7) by χ_ϵ . The

²¹Note that we are assuming that a worker cannot receive a Y -sector wage while searching if that worker is shirking on an X -sector job. This makes sense if, for example, *effort* is not observable and third-party verifiable but *physical presence* on the job site is, and a worker can search while physically at the X -sector job site but cannot produce Y -sector output while there. Thus, an X -sector employer would be able to sue to recover the wage just paid if the worker was absent, working another job, instead of on site at the location of the X firm.

first-order conditions with respect to ω_ϵ and \widetilde{W}_ϵ respectively are

$$-\pi_\epsilon - \psi_\epsilon + \pi_\epsilon \lambda \frac{\mu'(\frac{\omega_\epsilon}{P})}{P} + \chi_\epsilon = 0, \quad (9)$$

$$\rho\beta\pi_\epsilon\Omega'(\widetilde{W}_\epsilon) + \rho\beta\psi_\epsilon\Omega'(\widetilde{W}_\epsilon) + v_\epsilon + \rho\beta\pi_\epsilon\lambda = 0, \quad (10)$$

and in addition there is an envelope condition

$$\Omega'(W) + \lambda = 0. \quad (11)$$

Since $\Omega'(W) < 0$, for equation (11) to hold, we must have $\lambda > 0$, hence the target utility constraint always bind. Therefore, at the beginning of the employment relationship it is feasible for the employer to push the worker's payoff down to the opportunity payoff. Since it is in the interest of the employer to do so, it is clear that workers joining X -sector employment receive the same payoff that they would receive in the Y sector, $V^{WR} = V^{WS}$. From equation (1), this immediately tells us²²

$$V^{WS} = \frac{\mu(\frac{\omega^y}{P})}{1 - \beta}, \quad (12)$$

and condition (5)' can be rewritten as

$$\widetilde{W}_\epsilon \geq \widetilde{W}^*, \text{ where } \widetilde{W}^* \equiv \frac{\mu(\frac{\omega^y}{P})}{1 - \beta} + \frac{k}{\rho\beta}. \quad (5)''$$

To recapitulate, in each period the employer maximizes equation (3), subject to constraints (4), (5)'', (6), and (7). In the first period of the relationship, the worker's target utility $W = W_0$ is given by V^{WS} , but in the second period it is determined by the values of \widetilde{W}_ϵ chosen in the first period and by the first-period state, and similarly in later periods it is determined by choices made at earlier dates. We impose an assumption.

²²Of course, this implies that, in equilibrium, Y -sector workers are indifferent between searching and not searching, so if a small search cost were imposed, there would be no search (this is a version of the Diamond search paradox). However, this feature would disappear if any avenue were opened up to allow workers to capture some portion of X -sector rents. For example, for simplicity, we have assumed that employers have all of the bargaining power, but this could be relaxed. In addition, we have assumed that k is common knowledge, but it would be reasonable to assume that different workers have different values of k , and while employers know the distribution of this parameter, they do not know any given worker's value of it. Either of these modifications would very substantially increase the complexity of the model, but would give X -sector workers some portion of the rents and thus avoid the Diamond paradox.

Assumption 1. In the first period of an employment relationship, the employer’s incentive-compatibility constraint (4) does not bind in either state.

We will discuss sufficient conditions for this later (see Lemma 2 in Section 4). We are now ready to describe the equilibrium.

Proposition 1. In the first period of an equilibrium employment relationship, the wage is set equal to ω^y in each state and the continuation payoff for the worker in each state is set equal to \widetilde{W}^* . In the second period and all subsequent periods of the employment relationship, there is a pair of values ω_ϵ^* for $\epsilon = G, B$ such that regardless of history (provided neither partner has shirked), the wage is equal to ω_ϵ^* in state ϵ . In addition, the worker’s continuation payoff is always equal to \widetilde{W}^* . Further, after the first period there are three possible cases:

- (i) The employer’s incentive compatibility constraint (4) never binds, and $\omega_G^* = \omega_B^*$.
- (ii) The employer’s incentive compatibility constraint (4) binds in the bad states but not in the good states, and $\omega_G^* > \omega_B^*$.
- (iii) The employer’s incentive compatibility constraint (4) always binds, and $x_G - \omega_G^* = x_B - \omega_B^*$.

Proof. See appendix.

Two types of wage agreements are possible in equilibrium. In each type, (under Assumption 1) the worker goes through an ‘apprenticeship period’ at the beginning of the relationship in which Y -sector wage, ω^y , is paid. Thereafter, if the employer’s incentive constraint does not bind, the worker receives a constant wage $\omega_G^* = \omega_B^*$. We will call this type as *wage-smoothing* agreement. On the other hand, if the employer’s constraint ever binds, then it binds only (and always) in the bad state, resulting in state-dependent wages with $\omega_G^* > \omega_B^*$.²³ We will call this type as *fluctuating-wage* agreement. The key idea is that it is never optimal to promise more future utility than is required to satisfy the worker’s

²³The case in which the employer’s incentive constraint binds in both states occurs in a zero-measure portion of the parameter space, hence will be ignored.

incentive constraint (5)'', so after the first period of the relationship, the worker's target utility is always equal to \widetilde{W}^* (Thus, in the first period we have $W_{\min} = V^{WS}$, whereas in any subsequent period we have $W_{\min} = \widetilde{W}^*$). This means that after the first period, the optimal wage settings by the employer are stationary. We will analyze each type of agreements in turn.²⁴

3 Wage-Smoothing Agreement

In any type of equilibria (either wage smoothing or fluctuating wage), the worker's incentive compatibility constraint and the target utility constraint always bind (see the proof of Proposition 1 in the appendix). The former implies that $\widetilde{W}_\epsilon = \widetilde{W}^*$ for any ϵ , and after the first period, $W = \widetilde{W}^*$ and the latter implies that constraint (6) holds with equality. Moreover, under wage-smoothing equilibrium, the employer's incentive compatibility constraint is slack in both states, therefore paying a constant wage in both states is feasible. We can calculate this constant wage by substituting equations (12) and (5)'' into constraint (6)

$$\mu\left(\frac{\omega^*}{P}\right) = \mu\left(\frac{\omega^y}{P}\right) + \frac{k}{\rho\beta}, \quad (13)$$

where we denote the constant wage paid under wage-smoothing case with ω^* . We will name this as the 'efficiency wage.' It represents the lowest constant wage that can be given to the worker in a self-enforcing agreement. Given that employers are risk-neutral whereas workers are risk averse, employers always prefer wage smoothing, since it delivers the lowest expected wage payment to workers. However, wage-smoothing is not always possible since the employer's constraint may bind. If it binds, it does so only (and always) in the bad state. Hence, by computing the values of V^{ES} and $\Omega(\widetilde{W}^*)$ under wage-smoothing equilibrium, we

²⁴Note that in the current setup, all bargaining power is allocated to the employer, therefore we have $V^{WR} = V^{WS}$. Giving some bargaining power to the worker makes the employer's incentive compatibility constraint tighter and expands the portion of the parameter space in which wage volatility occurs. However, as long as the worker's bargaining power is not too large, our main insights continue to hold, but at the cost of greater complexity. More specifically, for any value of V^{WR} with $V^{WS} \leq V^{WR} < \widetilde{W}^*$, the expected wage in the first period will be lower than the expected wage in the second period and it is possible to find a region in the parameter space where the same types of wage contracts as in the current setup are offered in equilibrium.

can determine the conditions under which the employer's bad state incentive compatibility constraint (given in constraint (4) for $\epsilon = B$) is satisfied.

Now we are ready to find V^{ES}

$$V^{ES} = Q^E \rho \beta \left(\Omega(\widetilde{W}^*) + \omega^* - \omega^y \right) + Q^E (1 - \rho) \beta V^{ES} + (1 - Q^E) \beta V^{ES}. \quad (14)$$

In addition, $\Omega(\widetilde{W}^*)$ is given by

$$\Omega(\widetilde{W}^*) = \frac{\bar{x} - \omega^* + (1 - \rho) \beta V^{ES}}{1 - \rho \beta}. \quad (15)$$

If we substitute equation (15) into equation (14) and rearrange, we obtain

$$V^{ES} = \frac{Q^E \rho \beta [\bar{x} - \rho \beta \omega^* - (1 - \rho \beta) \omega^y]}{(1 - \beta) [1 - (1 - Q^E) \rho \beta]}. \quad (16)$$

Notice that V^{ES} is increasing in Q^E ($\frac{\partial V^{ES}}{\partial Q^E} > 0$) and decreasing in $\omega^y = p^y$ ($\frac{\partial V^{ES}}{\partial \omega^y} < 0$).

The employer's bad-state incentive compatibility constraint is given by

$$x_B - \omega^* + \rho \beta \Omega(\widetilde{W}^*) + (1 - \rho) \beta V^{ES} \geq V^{ES}. \quad (17)$$

Using equation (15), this becomes

$$x_B - \omega^* + \rho \beta \pi_G (x_G - x_B) \geq (1 - \beta) V^{ES}. \quad (18)$$

We can interpret inequality (18) as follows. Suppose that the employer's incentive constraint just binds in the bad state so that the employer's payoff is equal to V^{ES} in that state. The employer's average payoff is then equal to $\pi_G (V^{ES} + x_G - x_B) + (1 - \pi_G) V^{ES} = V^{ES} + \pi_G (x_G - x_B)$. From inequality (17), the employer's payoff in the bad state if she does not renege is $x_B - \omega^* + \rho \beta [V^{ES} + \pi_G (x_G - x_B)] + (1 - \rho) \beta V^{ES}$ and if she reneges is V^{ES} . Equating these two gives inequality (18) as an equality.

Next, substituting equation (16) into inequality (18) and rearranging, we obtain

$$\omega^* \leq \frac{Q^E \rho \beta \omega^y + x_B + (1 - Q^E) \rho \beta \pi_G (x_G - x_B)}{1 + Q^E \rho \beta}. \quad (19)$$

At this stage, it will be instructive to look at the limiting cases. As $Q^E \rightarrow 1$, wage smoothing is sustainable if and only if $x_B \geq \omega^* + \rho \beta (\omega^* - \omega^y)$. If an employer can immediately find

a new worker, renegeing in the bad state involves paying no wage and receiving no output now, and starting a new relationship with a new worker next period. The loss from doing so is the current output, x_B . The benefit is the current wage that is not paid to the worker, plus the gain from paying a lower wage next period because the new worker will be in her apprenticeship period. Notice that since new workers are cheaper than old ones, the employer still has a temptation to renege even if the worker's productivity in the bad state exceeds her wage, e.g., $x_B \geq \omega^*$ but $x_B < \omega^* + \rho\beta(\omega^* - \omega^y)$.

In the other limiting case where $Q^E \rightarrow 0$, wage smoothing is sustainable if and only if $x_B + \rho\beta\pi_G(x_G - x_B) \geq \omega^*$. Given that the employer cannot find another worker at all, the wage-smoothing equilibrium can be sustained even if the employer makes losses in the bad state, e.g., $x_B < \omega^*$ but $x_B + \rho\beta\pi_G(x_G - x_B) \geq \omega^*$. Recalling that ω^* is determined by parameters through equation (13), we assume the following.

Assumption 2. The bad-state output satisfies the following condition

$$\omega^* < x_B < \omega^* + \rho\beta(\omega^* - \omega^y).$$

This assumption ensures that it is socially optimal to produce in both good and bad states. It also guarantees that for a given value of $Q^E \in (0, 1)$, the parameter space is partitioned into two regions where the wage-smoothing and the fluctuating-wage equilibria take place. Later on, the expression in (19) will be useful to do this partition.

Next, we turn to those fluctuating-wage equilibria.

4 Fluctuating-Wage Agreement

To reiterate, in any type of equilibria, the worker's incentive compatibility constraint and the target utility constraint always binds. However, in a fluctuating-wage equilibrium, unlike in a wage-smoothing equilibrium, the employer's bad-state incentive constraint binds, implying that in the bad state, the employer cannot afford to pay the same high wage she pays in the good state. Accordingly, we can follow the same steps as before in deriving (13)

by substituting equations (12) and (5)'' into constraint (6) to obtain

$$E_\epsilon \mu\left(\frac{\omega_\epsilon^*}{P}\right) = \mu\left(\frac{\omega^y}{P}\right) + \frac{k}{\rho\beta}. \quad (20)$$

Equation (20) states that in any period after the first, the expected utility promised to an X -sector worker must be enough to compensate that worker next period, in expected value, for the current disutility of effort. This equation is represented in Figure 1 by the downward-sloping curve WW , which is strictly convex due to the worker's risk aversion.

[Insert Figure 1 here]

Under the fluctuating-wage equilibrium, the employer's binding bad-state incentive compatibility constraint is given by

$$x_B - \omega_B^* + \rho\beta\Omega(\widetilde{W}^*) + (1 - \rho)\beta V^{ES} = V^{ES}. \quad (21)$$

Developing expressions for $\Omega(\widetilde{W}^*)$ and V^{ES} by changing ω^* with $E_\epsilon \omega_\epsilon^*$ in equations (15) and (16), respectively and substituting them into equation (21) yields

$$\omega_B = \frac{-\rho\beta\pi_G\omega_G + Q^E\rho\beta\omega^y + x_B + (1 - Q^E)\rho\beta\pi_G(x_G - x_B)}{1 - \rho\beta(\pi_G - Q^E)}, \quad (22)$$

which is the straight downward-sloping line EE in Figure 1.

We can now summarize the equilibrium with the help of Figure 1. On the vertical axis we have the bad-state wage and on the horizontal axis we have the good-state wage. The EE line represents the employer's bad-state incentive compatibility constraint and the employer would not offer any wage combination above this line. The WW curve represents a combination of the worker's incentive compatibility constraint and the target utility constraint and the worker would not accept any wage combination below this curve. The efficiency wage, ω^* , is given by the intersection of WW with the 45°-line. Any movement along the WW curve toward that point increases the employer's profit.

In equilibrium, the employer will choose the wage combination that minimizes expected wages, subject to the two constraints and this amounts to choosing ω^* if it is on or below

EE , and choosing the intersection of EE and WW closest to the 45° -line otherwise. In this figure, by assumption, we focus on the fluctuating-wage equilibrium, so efficiency wage is unattainable as it is above the EE line. Therefore, we know that the intersection of EE with the 45° -line occurs below the intersection of WW with the 45° -line. Further, since we have shown that in equilibrium the good-state wage is never below the bad-state wage, the WW curve and EE line must intersect *below* the 45° -line. Given the concavity of WW and the linearity of EE , there will clearly be two such intersections, but the one that will be chosen by the employer is the one closest to the 45° -line, as shown, because it will offer the lowest expected wage consistent with the constraints. This means that at the point of intersection that determines ω_B and ω_G , EE is flatter than WW . As a result, it is clear that anything that shifts the EE line down without shifting WW will raise ω_G and lower ω_B . In addition, it is useful to note that, since the WW curve is a worker indifference curve, holding k constant, anything that shifts up the WW line (whether or not it shifts the EE line) raises worker welfare.

We are ready now to state the sufficient condition for Assumption 1 to hold.

Lemma 2. A sufficient condition for Assumption 1 to hold is $x_B \geq \omega^y + \rho\beta(E_\epsilon\omega_\epsilon^* - \omega^y)$.²⁵

Proof. See Appendix.

In what follows we will do comparative statics on X -sector wages by changing Q^E and p^y .

5 Comparative Statics Analysis

We will start our comparative statics by analyzing the effect of a rise in Q^E on X -sector wages and worker welfare, while keeping p^y constant. We will show that as it becomes easier for the employer to fill a vacancy, her temptation to cheat within a given relationship increases, which shifts the EE line down without shifting WW . Whether this shift has any

²⁵Note that there exists a portion of the parameter space where Assumption 1 and Assumption 2 are both satisfied. We will comment further on this in footnote 26 of Section 6.

effect on wages depend on what type of equilibria prevails, i.e., wage smoothing versus fluctuating wage. We can summarize our findings in the following proposition.

Proposition 2. An increase Q^E holding $\omega^y = p^y$ constant will have no effect on worker welfare. Under fluctuating-wage equilibrium, it will raise ω_G and lower ω_B , in the process raising average X -sector wages. Under wage-smoothing equilibrium, it will have no effect on wages if wage smoothing is still possible; otherwise, the fluctuating-wage equilibrium with wage volatility and rising expected wage payment results.

Proof. See Appendix.

An increase in Q^E makes it is easier for the employer to find a new worker. This in turn aggravates the employer's temptation to revoke on wage promises made to the seasoned worker, especially when profits are low. The increased temptation makes the employer's incentive constraint tighter, implying that a rise in Q^E will shift the EE down, without having any effect on WW (neither equation (13) in case of wage-smoothing equilibrium nor equation (20) in case of fluctuating-wage equilibrium will change). Since WW will not shift, worker welfare stays intact. Under the fluctuating-wage equilibrium, the downward shift of EE lowers the bad-state wage and to compensate the worker, raises the good-state wage. This raises the expected wage payment in the X sector due to worker's risk aversion. On the other hand, under wage smoothing, as long as the employer's bad-state incentive constraint does not bind, there will be no effect on wages. However, if it binds with an increase in Q^E , then it is not possible to sustain a constant wage, hence we move into a fluctuating-wage equilibrium with wage volatility and rising expected wage payment.

Consequently, Q^E affects the employer's well being in two ways. It has a positive direct effect given that it is easier for the employer to fill a vacancy and be productive. However, it may have a negative indirect effect such that in a given relationship with fluctuating wages, the employer's surplus is lower since the expected wage payment to the worker increases due to the increase in wage volatility.

Next, we analyze the effect of a change in p^y on X -sector wages and worker welfare.

Proposition 3. An increase in p^y always raises welfare of workers. Further, under fluctuating-wage equilibrium, an increase in p^y will raise ω_G and lower ω_B , in the process raising average X -sector real wages. Instead, under wage-smoothing equilibrium, an increase in p^y will raise the efficiency wage in real terms if wage smoothing is still possible; otherwise, the fluctuating-wage equilibrium with wage volatility and rising expected real wage payment results.

Proof. See appendix.

In particular, a rise in p^y will shift both curves upward. The WW curve shifts up because the worker's opportunity cost has risen. The EE curve shifts up because, for given wages (either ω_G and ω_B in a fluctuating-wage equilibrium or ω^* in wage-smoothing equilibrium) the rise in the worker's opportunity cost lowers the degree to which new workers are cheaper than incumbents (recall that a new worker is paid her opportunity wage ω^y in the first period of employment). The former tends to increase the wage volatility while the latter tends to decrease it. Overall, the former effect dominates since the higher opportunity cost of X -sector workers lowers the joint surplus available to worker-employer pair in the X sector and also lowers the share of the surplus that can be captured by the employer. This sharpens the employer's incentive-compatibility constraint. In other words, the rise in the worker's opportunity cost makes the employer more prone to cheat in the bad-profitability state.

As a result, if fluctuating-wage equilibrium prevails, X -sector wages become more volatile as p^y rises. This also raises expected wage payment in the X sector. On the other hand, under wage smoothing, as long as the employer's bad-state incentive constraint does not bind, an increase in p^y will raise the efficiency wage without causing any wage volatility. Nevertheless, if it binds with an increase in p^y , then it is not possible to sustain a constant wage, hence we move into a fluctuating-wage equilibrium with wage volatility and rising expected wage payment.

Note that when p^y rises, the real wage in the Y sector, $\frac{\omega^y}{P(p^y)}$, goes up (so does $\mu(\frac{\omega^y}{P(p^y)})$) since the elasticity of $P(p^y)$ with respect to p^y is less than 1. To satisfy either equation (20)

or equation (13), the expected wage (in case of fluctuating-wage equilibrium) or the efficiency wage (in case of wage-smoothing equilibrium) must rise more than p^y . This implies that the expected wage or the efficiency wage rises not only in nominal terms but also in real terms. In turn, the rise in real wage causes the WW curve to shift upwards and increases worker welfare.

Propositions 2 and 3 imply the following.

Corollary. For a given p^y , there is a value $Q_{VV}^E(p^y) \in [0, 1]$, such that if $Q^E < Q_{VV}^E(p^y)$ a wage-smoothing equilibrium can be sustained, while if $Q^E > Q_{VV}^E(p^y)$ it cannot. Further, $Q_{VV}^E(p^y)$ is decreasing in p^y .

Proof. See Appendix.

In Figure 2, the downward-sloping VV curve represents the function $Q_{VV}^E(p^y)$. On this curve, (Q^E, p^y) combinations are such that the employer's incentive compatibility constraint holds with equality, and thus forms a border between the wage-smoothing and the fluctuating-wage equilibria. To the left of VV curve, we have wage smoothing and to the right, we have fluctuating wages since it is not possible to sustain efficiency wage. Consequently, we show that for given parameters, wage smoothing is possible if it is sufficiently difficult for an employer to find a new worker (i.e., for a given p^y , when Q^E is low) or if Y -sector output is sufficiently cheap (i.e., for a given Q^E , when p^y is low).

[Insert Figure 2 here]

To see the effect of a change in Q^E and p^y , consider the part of the parameter space in Figure 2 where we have fluctuating-wage equilibrium. Any movement up and to the right from a point on or above the VV curve results in a rise in wage volatility. Moreover, while any upward movement improves workers' welfare in both sectors, any horizontal movement has no such effect.

Additionally, when (Q^E, p^y) combination is close to the VV curve, we are close to the efficiency wage region, thus ω_G^* is close to ω_B^* , so $x_G - \omega_G^* > x_B - \omega_B^*$. Increasing Q^E

while holding p^y constant increases wage volatility by increasing ω_G^* and decreasing ω_B^* . As we continue increasing Q^E either in the limit we reach $Q^E = 1$ with the inequality $x_G - \omega_G^* > x_B - \omega_B^*$ still holds, or there exists a value of $Q_{BB}^E(p^y)$ such that the employer's incentive compatibility constraint binds in both states, so $x_G - \omega_G^* = x_B - \omega_B^*$ and for any value of $Q^E > Q_{BB}^E(p^y)$ we will have $x_G - \omega_G^* < x_B - \omega_B^*$, where no equilibrium exists as shown in Proposition 1. The function $Q_{BB}^E(p^y)$ is demonstrated by the downward-sloping BB curve in Figure 2.

In the following section, we will analyze the general equilibrium where Q^E and p^y are endogenously determined.

6 General Equilibrium

We consider the steady state equilibrium where the amount of each good produced is equal to the amount of each good consumed. We first determine the equilibrium value of Q^E . We know that in any period, the total number of matches in the X sector, $\Phi(n, m, \phi)$, is a function of number of workers searching (n), number of employers searching (m) and the effectiveness of search technology (ϕ). Therefore, at the steady state, the probability of a searching employer to find a partner is given by $Q^E = \frac{\Phi(n, m, \phi)}{m} = \Phi(\frac{n}{m}, 1, \phi)$, an increasing function of $\frac{n}{m}$ and ϕ . Thus the steady state level of m must satisfy

$$m = \left(1 - \Phi\left(\frac{n}{m}, 1, \phi\right)\right) m + (1 - \rho)(E - m) + (1 - \rho)\Phi\left(\frac{n}{m}, 1, \phi\right)m.$$

On the right-hand side, the first term represents the number of employers with no match; the second term represents the number of previously-matched employers that are exogenously separated; and the last term represents the number of newly-matched employers that are immediately exogenously separated. A straightforward simplification of the above equation yields

$$m = E - \frac{\rho}{1 - \rho}\Phi\left(\frac{n}{m}, 1, \phi\right)m. \quad (23)$$

Derivation of the similar expression for the steady state number of workers results

$$n = L - \frac{\rho}{1 - \rho}\Phi\left(\frac{n}{m}, 1, \phi\right)m. \quad (24)$$

Using these two equations, we have the following proposition.

Proposition 4. The steady-state value of $\frac{n}{m}$ and Q^E is uniquely determined for a given value of ϕ . Therefore, we can write $Q^E(\phi)$. Moreover, $Q^E(\phi)$ is strictly increasing.

Proof. See Appendix.

An advancement in search technology affects the employer's steady state probability of finding a match both directly and indirectly. The direct effect, $\frac{\partial Q^E}{\partial \phi} = \frac{\partial \Phi(\frac{n}{m}, 1, \phi)}{\partial \phi} > 0$, increases the total number of matches in the X sector. The indirect effect, $\frac{\partial Q^E}{\partial(\frac{n}{m})} \frac{d(\frac{n}{m})}{d\phi} = \frac{\partial \Phi(\frac{n}{m}, 1, \phi)}{\partial(\frac{n}{m})} \frac{d(\frac{n}{m})}{d\phi}$, can be positive or negative depending on whether $E < L$ or $E > L$, respectively. However, as we show in the appendix, even if the indirect effect is negative, it is always the case that the direct effect dominates, and thus an improvement in the effectiveness of search technology makes it easier for an employer to fill a vacancy.

Next, we will determine the relative price of good Y , p^y . To that purpose, since consumers have identical and homothetic demands, it is sufficient to pin down the relative supply of good Y , r .

Proposition 5. The steady-state supply of X -sector output is an increasing function of ϕ , while the steady-state supply of Y -sector output is a decreasing function of ϕ . Therefore, the relative supply of Y -sector output, r , is a decreasing function ϕ , and the relative price of Y -sector output, p^y , is an increasing function of ϕ .

Proof. See appendix.

We can illustrate Propositions 4 and 5 with the help of Figure 3. It is the same as Figure 1 with the addition of the upward-sloping curve PP , which is the locus of market-clearing values that complete the general equilibrium. It gives the combinations of Q^E and p^y obtained by varying ϕ over the positive real line. More precisely, for a given value of ϕ , we can find the steady-state value of Q^E (as in Proposition 4) and the steady-state value

of the equilibrium relative price p^y (as in Proposition 5). Accordingly, as ϕ increases, the steady-state values of Q^E and p^y both go up.²⁶

[Insert Figure 3 here]

Note that the steepness of PP curve depends on elasticity of substitution implied by the utility function U between goods X and Y . Specifically, if the elasticity of substitution is high, then a given rise in ϕ and consequent drop in r will require only a small change in the relative price p^y to restore market clearing. Conversely, a low elasticity of substitution will require a large movement in p^y . For this reason, if they are very close substitutes, PP is arbitrarily flat, while if they are close to the case of perfect complementarity, it is arbitrarily steep.

Finally, we have all the tools to analyze the overall effect of a change in the effectiveness of search technology on equilibrium.

Theorem. An improvement in the effectiveness of search technology, i.e., a rise in ϕ , will raise the (average) real wage in both sectors, while raising the (expected) real wages and worker welfare in the whole economy. It also raises the wage volatility in the X sector if the new equilibrium has fluctuating wages. Besides, depending on the parameter values, it may also increase economy-wide wage volatility.

Proof. See Appendix.

The rise in search effectiveness, ϕ , has two effects on the steady-state equilibrium. First, it increases the number of productive employers by increasing their probability of finding a worker (Proposition 4). This aggravates the employer's temptation to shirk on wage promise

²⁶We can now specify the portion of the parameter space where both Assumptions 1 and 2 are satisfied. First note that for the wage-smoothing agreement, the wage-smoothing condition is strictly stronger than Assumption 1 since $\omega^* > \omega^y$. Hence, for the whole length of PP curve to the left of VV and for at least a segment of positive length to the right of VV , Assumption 1 will be satisfied. If it is also true that Assumption 2 holds at the intersection of PP and VV , then there is a segment of PP including its intersection with VV plus some distance on both sides in which Assumptions 1 and 2 are both satisfied. We assume this and restrict our attention to that segment.

and increases (or does not affect if efficiency wage is still sustainable) the expected wage and X -sector wage volatility (Proposition 2). This is the direct effect. In addition, there is also an indirect effect such that as more matched pairs occur and increases the amount of good X , less workers will be left to be employed in the Y sector, causing Y -sector good production to go down while raising its relative price, p^y (Proposition 5). In turn, from Proposition 3, a rise in p^y also aggravates the employer's temptation to shirk and increases the expected wage (or the efficiency wage if it is still attainable) in both real and nominal terms and increases X -sector wage volatility (or does not affect it if efficiency wage is still sustainable). Overall, we can conclude that as search effectiveness improves, in the X sector, expected wage (or efficiency wage) increases both in nominal and real terms whereas wage volatility increases only if fluctuating wage prevails in the new equilibrium.

We can also see what happens to economy-wide wage dynamics. First, we focus on expected real wages. As ϕ rises, we know that $p^y = \omega^y$ increases. We also know that elasticity of $P(p^y)$ with respect to p^y is less than 1, since this elasticity measure also represents consumption share of good Y . These two observations imply that workers that are employed in the Y sector and those that are in the apprenticeship period of their X -sector employment will experience an increase in their real wage. Further, for those incumbent workers in the X sector, a rise in ϕ increases the expected wage (or the efficiency wage in case of wage smoothing) more than the increase in p^y as can be seen from equation (20) (or from equation (13) in case of wage smoothing). Hence, their (expected) real wage increases even more than the former group of workers. Since any worker is employed either in the X sector or Y sector, this automatically implies economy-wide increase in (expected) real wages and worker welfare. Next, we turn our attention to economy-wide real wage volatility.²⁷ Once ϕ rises, there are two types of effects on real wage volatility which we call with a slight abuse of terminology as 'price' effect and 'compositional' effect.²⁸ The 'price' effect measures the

²⁷Our discussion is on real wage volatility, however nominal wage volatility can be directly inferred from it.

²⁸See Autor, Katz and Kerney (2008). There are opposing views in the literature regarding the relative importance of these two effects. Lemieux (2006) finds that most of the residual (within group) wage dispersion observed from 1973 to 2003 is due to compositional effect, whereas Autor, Katz and Kerney (2008) find

change in wage volatility while holding industrial composition intact, i.e., keeping the number of workers in each sector constant. Instead, the ‘compositional’ effect measures the change in wage volatility in response to changes in sectoral compositions only, i.e., the movement of workers between sectors with different degrees of wage volatility. We show that the former effect is always positive (i.e., increases wage volatility), whereas the latter can be either positive or negative. Therefore, if the ‘compositional’ effect is positive, we can conclude that economy-wide real wage volatility increases; otherwise, it depends on how strong is the ‘price’ effect vis-à-vis the ‘compositional’ effect.

7 Conclusion

In this paper, we employ a stylized model to analyze the relationship between market thickness, labor market flexibility and wage dynamics. We have shown that a rise in market thickness caused by advances in search technology can generate wage volatility by increasing labor market flexibility. Our findings indicate that improvements in search efficiency increases the average real wage and (potentially) wage volatility in the risky sector while raising the expected real wages and worker welfare in the whole economy. Moreover, it may also raise economy-wide real wage volatility. As a result, within this simple framework we manage to explain the transitory variation in workers’ earnings observed during 1970s and 1980s.

Our results have also some implications regarding welfare of workers. At the outset, it may seem that the greater instability of earnings caused by transitory shocks would decrease the welfare of risk-averse agents. However, this conclusion is premature and in order to reach a convincing verdict we need to determine how insurable these shocks are. When it is possible to insure against earnings volatility via risk-sharing arrangements, this negative welfare effect may not be a concern. This is what we have found in this paper; following an advancement in search technology, even if wage volatility rises, worker welfare does not decrease but rather rises due to relative-price effects. This result is in harmony with the

that the price effect remains a key force in explaining residual wage inequality between 1989 to 2005.

statement made by Edmund Phelps while discussing the work of Gottschalk, Moffitt, Katz and Dickens (1994):

“Insofar as increased transitory variance reflects wage flexibility, it means that labor markets are working more efficiently, which should be as welcome as increased price flexibility. Furthermore, individuals can take measures to soften the impact of transitory losses, and the welfare state offers additional insurance. In my view, efforts to make incomes more secure and insulate individuals from market signals would be the wrong response.” – Edmund Phelps, a general discussion on Gottschalk and Moffitt, Katz and Dickens (1994).

Appendix

Proof of Lemma 1. In the first-period of the employment relationship, the employer must give the worker at least as much as the worker’s outside option, otherwise she will never accept employment, thus, $V^{WR} \geq V^{WS}$. If we substitute V^{WS} for V^{WR} in equation (1), we obtain

$$V^{WS} \geq \mu\left(\frac{\omega^y}{P}\right) + Q^W \rho \beta V^{WS} + Q^W (1 - \rho) \beta V^{WS} + (1 - Q^W) \beta V^{WS}, \text{ or}$$

$$V^{WS} \geq \frac{\mu\left(\frac{\omega^y}{P}\right)}{1 - \beta} > 0. \quad (25)$$

Next, using equations (5)’ and (25), we have

$$\widetilde{W}_\epsilon \geq \widetilde{W}^* = V^{WS} + \frac{(1 - \beta) V^{WS} - \mu\left(\frac{\omega^y}{P}\right) + k}{\rho \beta} > V^{WS} > 0. \blacksquare$$

Proof of Proposition 1. In accordance with Assumption 1, when solving the first period problem, we will assume that the employer’s incentive compatibility constraint does not bind. A sufficient condition for this is provided in Lemma 2 in the main text.

Consider the first period problem. From the envelope condition given in equation (11), we know that the target utility constraint binds, so $\lambda > 0$ and $V^{WR} = V^{WS}$. Suppose that

the worker's incentive-compatibility constraint does not bind in state ϵ in the first period. Then $v_\epsilon = 0$, and since $\psi_\epsilon = 0$ because of Assumption 1, equation (10) becomes

$$\Omega'(\widetilde{W}_\epsilon) + \lambda = 0.$$

The envelope condition given in equation (11) implies that in the first period we have $\Omega'(W_0) + \lambda = 0$. This means $\widetilde{W}_\epsilon = W_0 = V^{WS}$. But since $V^{WS} < \widetilde{W}^*$ as shown in Lemma 1, this implies that the worker's incentive compatibility constraint (5)'' will be violated, a contradiction. Therefore, the worker's incentive compatibility constraint must bind in each state, ensuring that $\widetilde{W}_\epsilon = \widetilde{W}^*$. Given that $\widetilde{W}_\epsilon = \widetilde{W}^*$ and $W_0 = V^{WS}$, the target utility constraint (6) in the first period becomes

$$\sum_{\epsilon=G,B} \pi_\epsilon \mu\left(\frac{\omega_\epsilon}{P}\right) = \mu\left(\frac{\omega^y}{P}\right). \quad (26)$$

In addition, condition (9), with $\psi_\epsilon = 0$, becomes

$$-\pi_\epsilon + \pi_\epsilon \lambda \frac{\mu'\left(\frac{\omega_\epsilon}{P}\right)}{P} + \chi_\epsilon = 0.$$

If $\chi_\epsilon > 0$ for some ϵ , then $\omega_\epsilon = 0$. This clearly cannot be true for both values of ϵ , because that would imply a permanent zero wage, and it would not be possible to satisfy equation (26). Therefore, for at most one state, say ϵ' , $\chi_{\epsilon'} > 0$. Denote by ϵ'' the state with $\chi_{\epsilon''} = 0$. Then $\frac{\mu'(0)}{P} = \frac{1}{\lambda} \left(1 - \frac{\chi_{\epsilon'}}{\pi_{\epsilon'}}\right) < \frac{1}{\lambda} = \frac{\mu'\left(\frac{\omega_{\epsilon''}}{P}\right)}{P}$. However, given that $\omega_{\epsilon''}$ is non-negative and μ is strictly concave, this is impossible. We conclude that $\chi_\epsilon = 0$ in both states, and therefore $\omega_G = \omega_B = \omega^y$. Therefore, ω^y is the minimum first period wage required to make the worker willing to accept the job.

Consider now the second-period problem. As before due to the envelope condition, the target utility constraint binds. We know that the target continuation payoff for the worker is \widetilde{W}^* . We claim that the choice of next-period continuation payoff \widetilde{W}_ϵ will be equal to \widetilde{W}^* for $\epsilon = G, B$. If $v_\epsilon > 0$, then complementary slackness implies that $\widetilde{W}_\epsilon = \widetilde{W}^*$. Therefore, suppose that $v_\epsilon = 0$. This implies that condition (10) becomes

$$\Omega'(\widetilde{W}_\epsilon) = -\lambda \frac{\pi_\epsilon}{\pi_\epsilon + \psi_\epsilon}.$$

Since, by the envelope condition, $-\lambda = \Omega'(W)$, and as we recall for the second-period problem the worker's target utility $W = \widetilde{W}^*$, this becomes

$$\Omega'(\widetilde{W}_\epsilon) = \Omega'(\widetilde{W}^*) \frac{\pi_\epsilon}{\pi_\epsilon + \psi_\epsilon}. \quad (27)$$

If $\psi_\epsilon = 0$, this implies through the strict concavity of Ω that $W = \widetilde{W}^*$, and we are done. On the other hand, if $\psi_\epsilon > 0$, equation (27) then implies that $0 > \Omega'(\widetilde{W}_\epsilon) > \Omega'(\widetilde{W}^*)$, implying that $\widetilde{W}_\epsilon < \widetilde{W}^*$. However, this violates constraint (5)'. Therefore, all possibilities either imply that $W = \widetilde{W}^*$ or lead to a contradiction, and the claim is proven.

Since $W = \widetilde{W}^*$, the optimization problem in the third period of the relationship is identical to that of the second period. By induction, the target utility for the worker in every period after the first, regardless of history, is equal to \widetilde{W}^* , and so the wage chosen for each state in every period after the first, regardless of history, is the same.

Now, to establish the three possible outcomes, we consider each possible case in turn. Consider the optimization problem (3) at any date after the first period of relationship. First, suppose that the employer's constraint does not bind in either state. In this case, $\psi_\epsilon = 0$ for $\epsilon = G, B$. Condition (9) now becomes

$$-\pi_\epsilon + \pi_\epsilon \lambda \frac{\mu'(\frac{\omega_\epsilon}{P})}{P} + \chi_\epsilon = 0.$$

If $\chi_\epsilon > 0$ for some ϵ , then $\omega_\epsilon = 0$. This clearly cannot be true for both values of ϵ , because that would imply a permanent zero wage, and it would not be possible to satisfy constraint (6). (To see this, formally, substitute $W = \widetilde{W}_\epsilon = \widetilde{W}^*$, the expression for V^{WS} , and $\omega_G = \omega_B = 0$ into constraint (6), and note that the constraint is violated.) Therefore, for at most one state, say ϵ' , $\chi_{\epsilon'} > 0$. Denote by ϵ'' the state with $\chi_{\epsilon''} = 0$. Then $\frac{\mu'(0)}{P} = \frac{1}{\lambda} \left(1 - \frac{\chi_{\epsilon'}}{\pi_{\epsilon'}}\right) < \frac{1}{\lambda} = \frac{\mu'(\frac{\omega_{\epsilon''}}{P})}{P}$. However, given that $\omega_{\epsilon''}$ is non-negative and μ is strictly concave, this is impossible. We conclude that $\chi_\epsilon = 0$ in both states, and therefore $\omega_G = \omega_B$.

Next, suppose that we have $\psi_G > 0$ and $\psi_B = 0$, so that the employer's constraint binds only in the good state. We will show that this leads to a contradiction. Recall from the previous discussion that $\widetilde{W}_\epsilon = \widetilde{W}^*$ for both states, and note that, by assumption, constraint (4) is satisfied with equality for $\epsilon = G$. Since $x_B < x_G$, we now see that constraint (4) must

be violated for $\epsilon = B$ if $\omega_G \leq \omega_B$. Therefore, $\omega_G > \omega_B \geq 0$. This implies that $\chi_G = 0$. Applying condition (9), then, we have

$$\frac{\mu'(\frac{\omega_G}{P})}{P} = \frac{1}{\lambda} \left(1 + \frac{\psi_G}{\pi_G} \right) > \frac{1}{\lambda} \left(1 - \frac{\chi_B}{\pi_B} \right) = \frac{\mu'(\frac{\omega_B}{P})}{P},$$

which contradicts the requirement that $\omega_G > \omega_B$. This shows that it is not possible for the employer's constraint to bind only in the good state.

Now suppose that we have $\psi_G = 0$ and $\psi_B > 0$, so that the employer's constraint binds only in the bad state. We now wish to prove that in this case $\omega_G > \omega_B$. Suppose to the contrary that $\omega_G \leq \omega_B$. This implies that $\omega_B > 0$ (since, as shown earlier, it is not possible to have zero wage in both states), so that $\chi_B = 0$. Then, from condition (9)

$$\frac{\mu'(\frac{\omega_B}{P})}{P} = \frac{1}{\lambda} \left(1 + \frac{\psi_B}{\pi_B} \right) > \frac{1}{\lambda} \left(1 - \frac{\chi_G}{\pi_G} \right) = \frac{\mu'(\frac{\omega_G}{P})}{P},$$

which implies that $\omega_G > \omega_B$. Therefore, we have a contradiction, and we conclude that $\omega_G > \omega_B$.

Finally, suppose that the employer's constraint binds in both states. Given that $\widetilde{W}_\epsilon = \widetilde{W}^*$ in both states, equality in both states for constraint (4) requires that short-term profits $x_\epsilon - \omega_\epsilon^*$ are equal in the two states.

We have thus eliminated all possibilities aside from those listed in the statement of the proposition. ■

Proof of Lemma 2. First note that under wage-smoothing agreement since the employer's incentive compatibility does not bind in the second period and thereafter, it will not bind in the first period as well since $\omega^* > \omega^y$. Therefore, we need to focus on the fluctuating-wage agreement where the employer's bad-state incentive compatibility constraint binds in the second and subsequent periods. In the first period of a fluctuating-wage agreement, for the employer's bad-state incentive compatibility constraint to be slack, we need

$$x_B - \omega^y + \rho\beta\Omega(\widetilde{W}^*) + (1 - \rho)\beta V^{ES} \geq V^{ES}.$$

Using the expressions for $\Omega(\widetilde{W}^*)$ and V^{ES} by changing ω^* with $E_\epsilon\omega_\epsilon^*$ in equations (15) and

(16), respectively and substituting them into above equation, we obtain

$$x_B - [\omega^y + \rho\beta(E_\epsilon\omega_\epsilon^* - \omega^y)] + (1 - Q^E)\rho\beta\pi_G(x_G - x_B) \geq 0. \quad (28)$$

The last term is positive. Therefore, for condition (28) to hold, it is sufficient to have $x_B \geq \omega^y + \rho\beta(E_\epsilon\omega_\epsilon^* - \omega^y)$. ■

Proof of Proposition 2. Consider the fluctuating-wage equilibrium. Differentiating equation (22) with respect Q^E while holding $\omega^y = p^y$ constant, we obtain

$$\frac{d\omega_B}{dQ^E} \Big|_{dp^y=0} = -\rho\beta \frac{x_B - [\rho\beta\pi_G\omega_G + (1 - \rho\beta\pi_G)\omega^y] + [1 + \rho\beta(1 - \pi_G)]\pi_G(x_G - x_B)}{[1 - \rho\beta(\pi_G - Q^E)]^2}.$$

Therefore,

$$\frac{d\omega_B}{dQ^E} \Big|_{dp^y=0} < 0 \Leftrightarrow x_B - [\rho\beta\pi_G\omega_G + (1 - \rho\beta\pi_G)\omega^y] + [1 + \rho\beta(1 - \pi_G)]\pi_G(x_G - x_B) > 0. \quad (29)$$

We can rewrite the condition in (29) as

$$\left. \begin{aligned} & [x_B - [\omega^y + \rho\beta(E_\epsilon\omega_\epsilon^* - \omega^y)] + (1 - Q^E)\rho\beta\pi_G(x_G - x_B)] \\ & + [1 - \rho\beta(\pi_G - Q^E)]\pi_G(x_G - x_B) + \rho\beta\pi_B(\omega_B - \omega^y) \end{aligned} \right\} > 0.$$

Notice that the expression in the first line is the same expression given in condition (28) (which is necessary and sufficient for Assumption 1 to hold) and it is non-negative. The expression in the second line needs to be positive since the first term is positive and the second term is non-negative since $\omega_B \geq \omega^y$. The latter condition needs to be true otherwise it would not be possible to pay ω^y in the first period. Therefore, the above equation must be positive and this proves that $\frac{d\omega_B}{dQ^E} \Big|_{dp^y=0} < 0$.

Now, consider equation (20). Using this equation, we can find

$$\frac{d\omega_G}{d\omega_B} = -\frac{1 - \pi_G}{\pi_G} \frac{\mu'(\frac{\omega_B}{P})}{\mu'(\frac{\omega_G}{P})} < 0.$$

Since $\frac{d\omega_B}{dQ^E} \Big|_{dp^y=0} < 0$ and $\frac{d\omega_G}{d\omega_B} < 0$, we must have $\frac{d\omega_G}{dQ^E} \Big|_{dp^y=0} > 0$. Given the strict concavity of μ , expected wage payment in the X sector must rise, i.e., $\frac{dE_\epsilon\omega_\epsilon}{dQ^E} \Big|_{dp^y=0} > 0$, for equation (20) to hold.

In the case of wage smoothing, differentiating the right-hand side (*RHS*) of expression (19) with respect Q^E while holding $\omega^y = p^y$ constant, we obtain

$$\frac{dRHS}{dQ^E} \Big|_{dp^y=0} = -\rho\beta \frac{x_B - \omega^y + (1 + \rho\beta) \pi_G(x_G - x_B)}{(1 + Q^E \rho\beta)^2} < 0.$$

The inequality that $x_B - \omega^y + (1 + \rho\beta) \pi_G(x_G - x_B) > 0$ follows since condition (28) must be true for Assumption 1 to hold. This means that as Q^E increases the employer's bad state incentive constraint becomes tighter, but it will have no effect on the efficiency wage, ω^* , as long as the constraint does not bind. On the other hand, if the constraint starts to bind once Q^E rises, then we are in the fluctuating-wage zone, so wage volatility as well as expected wage payment increases in the X sector. ■

Proof of Proposition 3. First consider the fluctuating-wage equilibrium. Totally differentiating the WW curve given in equation (20) with respect to p^y and recalling that $\omega^y = p^y$, we obtain

$$\frac{\pi_G \mu'(\frac{\omega_G}{P}) \frac{d\omega_G}{dp^y} + \pi_B \mu'(\frac{\omega_B}{P}) \frac{d\omega_B}{dp^y}}{P} - \frac{[\pi_G \omega_G \mu'(\frac{\omega_G}{P}) + \pi_B \omega_B \mu'(\frac{\omega_B}{P})] P'}{P^2} = \frac{(P - p^y P') \mu'(\frac{p^y}{P})}{P^2}. \quad (30)$$

Now, using Roy's identity, for any consumer (either employer or worker) we can get the Marshallian demand for good Y as $c^Y(p^y, I) = \frac{p^y I}{P}$. Therefore, good Y 's share in total consumer expenditure is $\frac{p^y c^Y}{I} = \frac{p^y P'}{P} = \alpha^y$. Hence, equation (30) can be rewritten as

$$\pi_G \mu'(\frac{\omega_G}{P}) \frac{d\omega_G}{dp^y} + (1 - \pi_G) \mu'(\frac{\omega_B}{P}) \frac{d\omega_B}{dp^y} = \frac{\alpha^y}{p^y} \left[\pi_G \omega_G \mu'(\frac{\omega_G}{P}) + \pi_B \omega_B \mu'(\frac{\omega_B}{P}) \right] + (1 - \alpha^y) \mu'(\frac{p^y}{P}).$$

Similarly, totally differentiating the EE line given in equation (22) with respect to p^y and recalling that $\omega^y = p^y$, we obtain

$$\rho\beta\pi_G \frac{d\omega_G}{dp^y} + [1 - \rho\beta(\pi_G - Q^E)] \frac{d\omega_B}{dp^y} = Q^E \rho\beta. \quad (31)$$

Equations (30) and (31) are then a system of two linear equations in two unknowns, $\frac{d\omega_G}{dp^y}$ and $\frac{d\omega_B}{dp^y}$. Solving for $\frac{d\omega_B}{dp^y}$, we obtain

$$\frac{d\omega_B}{dp^y} = -\rho\beta\pi_G \frac{\alpha^y \frac{\pi_G \omega_G \mu'(\frac{\omega_G}{P}) + \pi_B \omega_B \mu'(\frac{\omega_B}{P})}{\omega^y} + (1 - \alpha^y) \mu'(\frac{p^y}{P}) - Q^E \mu'(\frac{\omega_G}{P})}{D},$$

where $D \equiv (1 - \pi_G) \mu'(\frac{\omega_B}{P}) [1 - \rho\beta(\pi_G - Q^E)] \left[\frac{\pi_G}{1 - \pi_G} \frac{\mu'(\frac{\omega_G}{P})}{\mu'(\frac{\omega_B}{P})} - \frac{\rho\beta\pi_G}{1 - \rho\beta(\pi_G - Q^E)} \right]$ is the determinant of the system. The term in the last bracket represents the difference (in absolute value) between the slope of WW curve and EE line. Since at the equilibrium the WW curve is steeper than the EE line, it is positive. Hence, $D > 0$. Note that

$$\frac{\pi_G \omega_G \mu'(\frac{\omega_G}{P}) + (1 - \pi_G) \omega_B \mu'(\frac{\omega_B}{P})}{\omega^y} > \frac{\pi_G \omega_G \mu'(\frac{\omega_G}{P}) + (1 - \pi_G) \omega_B \mu'(\frac{\omega_B}{P})}{\pi_G \omega_G + (1 - \pi_G) \omega_B} > \mu'(\frac{\omega_G}{P}).$$

The first inequality holds because the condition defining the WW curve implies that $\omega^y < \pi_G \omega_G + (1 - \pi_G) \omega_B$, and the second holds because the middle expression is a weighted average of $\mu'(\frac{\omega_G}{P})$ and $\mu'(\frac{\omega_B}{P})$, of which the former is smaller. This implies that

$$\alpha^y \frac{\pi_G \omega_G \mu'(\frac{\omega_G}{P}) + \pi_B \omega_B \mu'(\frac{\omega_B}{P})}{\omega^y} + (1 - \alpha^y) \mu'(\frac{p^y}{P}) > \mu'(\frac{\omega_G}{P}) > Q^E \mu'(\frac{\omega_G}{P}), \text{ so}$$

$$\frac{d\omega_B}{dp^y} < 0.$$

Since $\frac{d\omega_B}{dp^y} < 0$, equation (31) requires that $\frac{d\omega_G}{dp^y} > 0$, and therefore $\frac{d(\omega_G - \omega_B)}{dp^y} > 0$. Moreover, it is easy to see that $\frac{d(\frac{p^y}{P(p^y)})}{dp^y} = \frac{1 - \alpha^y}{P(p^y)} > 0$. Therefore, given the strict concavity of μ , we must have $\frac{dE_\epsilon \omega_\epsilon}{dp^y} > 1$, for equation (20) to hold.

Now consider the wage-smoothing equilibrium. If after a rise in p^y , it is not possible to sustain wage smoothing anymore, then all the results derived above for the fluctuating-wage equilibrium hold. Conversely, if it is still possible to have wage smoothing after a rise in p^y , by totally differentiating equation (13) with respect to p^y and recalling that $\omega^y = p^y$, we obtain

$$\frac{\mu'(\frac{\omega^*}{P}) \frac{d\omega^*}{dp^y}}{P} - \frac{\omega^* P' \mu'(\frac{\omega^*}{P})}{P^2} = \frac{(P - p^y P') \mu'(\frac{\omega^y}{P})}{P^2}$$

Hence, we have

$$\frac{d\omega^*}{dp^y} = \alpha^y \frac{\omega^*}{\omega^y} + (1 - \alpha^y) \frac{\mu'(\frac{\omega^y}{P})}{\mu'(\frac{\omega^*}{P})} > 1, \quad (32)$$

since μ is strictly concave and $\omega^* > \omega^y$. ■

Proof of Corollary. For a given p^y , $Q_{VV}^E(p^y)$ can be defined as the value of Q^E that makes

inequality (19) hold with equality,

$$Q_{VV}^E(p^y) = \frac{x_B - \omega^* + \rho\beta\pi_G(x_G - x_B)}{\rho\beta[\pi_G(x_G - x_B) + \omega^* - p^y]}, \quad (33)$$

where we use $\omega^y = p^y$. Furthermore, under Assumption 2, we have $0 < Q_{VV}^E(p^y) < 1$.

Given equation (32), it is straightforward to verify that $\frac{dQ_{VV}^E(p^y)}{dp^y} < 0$. ■

Proof of Proposition 4. The number of matched employers is equal to $E - m$, and the number of matched workers is equal to $L - n$. These must always be equal, so

$$E - L = m - n. \quad (34)$$

Assume initially that $E > L$. Dividing equation (23) by equation (24) and rearranging, we obtain

$$\begin{aligned} \frac{E}{L} &= 1 + \frac{m - n}{\frac{\rho}{1-\rho}\Phi(\frac{n}{m}, 1, \phi)m + n}, \text{ or} \\ \frac{E}{L} &= 1 + \frac{1 - \frac{n}{m}}{\frac{\rho}{1-\rho}\Phi(\frac{n}{m}, 1, \phi) + \frac{n}{m}}. \end{aligned} \quad (35)$$

Since $E > L$, $\frac{n}{m} < 1$ must hold for the right-hand side of equation (35) to be greater than unity. Therefore, at an equilibrium, the right-hand side of equation (35) is strictly decreasing in $\frac{n}{m}$, so the equilibrium level of $\frac{n}{m}$ is uniquely determined for given values of $\frac{E}{L}$, ϕ and ρ .

Now consider the case where $E < L$. We can instead rewrite equation (35) as

$$\frac{L}{E} = 1 + \frac{1 - \frac{m}{n}}{\frac{\rho}{1-\rho}\Phi(1, \frac{m}{n}, \phi) + \frac{m}{n}}. \quad (36)$$

Since $E < L$, for the right-hand side of equation (36) to be greater than unity, $\frac{m}{n} < 1$ (or alternatively, $\frac{n}{m} > 1$) must hold. Therefore, at an equilibrium, the right-hand side of equation (36) is strictly increasing in $\frac{n}{m}$, so the equilibrium level of $\frac{n}{m}$ is uniquely determined for given values of other parameters.

Two observations are in order. First, from equation (34), we have $dm = dn$, for given values of E and L . Second, for given values of E , L and ρ , if $E > L$, then $\frac{n}{m}$ is a locally decreasing function of ϕ with $\frac{n}{m} < 1$, whereas if $E < L$, then $\frac{n}{m}$ is a locally increasing function of ϕ with $\frac{n}{m} > 1$. Then, these two observations imply that $\frac{dm}{d\phi} = \frac{dn}{d\phi} < 0$.

Now, we are ready to show that $Q^E(\phi)$ is strictly increasing. Using $Q^E(\phi) = \frac{\Phi(n(\phi), m(\phi), \phi)}{m(\phi)} = \Phi(\frac{n}{m}(\phi), 1, \phi)$, we can rewrite equation (23) as

$$E = m(\phi) \left[1 + \frac{\rho}{1 - \rho} Q^E(\phi) \right]. \quad (37)$$

Totally differentiating equation (37) with respect to ϕ yields

$$\begin{aligned} \frac{dQ^E}{d\phi} &= -\frac{1 - \rho}{\rho} \frac{m'(\phi)}{m(\phi)} \left[1 + \frac{\rho}{1 - \rho} Q^E(\phi) \right], \text{ or} \\ &= -\frac{1 - \rho}{\rho} \frac{m'(\phi)}{[m(\phi)]^2} E > 0, \text{ since } m'(\phi) < 0. \blacksquare \end{aligned}$$

Proof of Proposition 5. The number of employers producing X -sector output at time t is given by

$$E - m_t = \rho [E - m_{t-1} + \Phi(n_{t-1}, m_{t-1}, \phi)], \quad (38)$$

where on the right-hand side, the first term represents the number of previously-matched employers that are not exogenously separated at time t , and the second term represents the number of newly-matched employers that are not immediately exogenously separated.

Denote the aggregate X -sector output produced in period t by x_t . Since the average X -sector output of an operating firm in any period is given by \bar{x} , the number of employers producing X -sector output at time t must be also equal to $\frac{x_t}{\bar{x}}$. Thus, equation (38) becomes

$$\frac{x_t}{\bar{x}} = \rho \left[\frac{x_{t-1}}{\bar{x}} + \Phi(n_{t-1}, m_{t-1}, \phi) \right].$$

Furthermore, evaluating the above equation at the steady state, where $x_t = x_{ss}$, $n_t = n$, $m_t = m$, we obtain

$$\begin{aligned} \frac{x_{ss}}{\bar{x}} &= \frac{\rho}{1 - \rho} \Phi(n, m, \phi), \text{ or} \\ &= \frac{\rho}{1 - \rho} \Phi\left(L - \frac{x_{ss}}{\bar{x}}, E - \frac{x_{ss}}{\bar{x}}, \phi\right). \end{aligned}$$

Since Φ is a linear homogeneous in n and m , dividing each side by $\frac{x_{ss}}{\bar{x}}$ yields

$$\Phi\left(\frac{\bar{x}}{x_{ss}}L - 1, \frac{\bar{x}}{x_{ss}}E - 1, \phi\right) = \frac{1 - \rho}{\rho}.$$

From the above equation, it is easy to see that for given values of E , L , and ρ , $x_{ss}(\phi)$ is an increasing function of ϕ .

Now we return to Y -sector output. In period t , it is given by

$$\begin{aligned} y_t &= L - \frac{x_t}{\bar{x}}, \\ &= n_t. \end{aligned}$$

In the steady state, this becomes

$$\begin{aligned} y_{ss}(\phi) &= L - \frac{x_{ss}(\phi)}{\bar{x}}, \text{ or} \\ &= n(\phi). \end{aligned}$$

Since $x'_{ss}(\phi) > 0$ (alternatively, $n'(\phi) < 0$), $y_{ss}(\phi)$ is an decreasing function of ϕ . Hence, the ratio of Y -sector production to X -sector production, $r(\phi) = \frac{y_{ss}(\phi)}{x_{ss}(\phi)}$, is strictly decreasing function of ϕ .

Goods market clearing together with utility maximization condition implies $p^y(\phi) = \frac{U_2(1,r(\phi))}{U_1(1,r(\phi))}$. Since p^y is strictly decreasing in r and r is strictly decreasing in ϕ , p^y must be strictly increasing in ϕ . ■

Proof of Theorem. The first part of the theorem follows from Propositions 2, 3, 4, and 5. Therefore, consider the second part regarding the economy-wide wage volatility. Before analyzing the economy-wide real wage volatility, it will be helpful to derive the following.

$$\begin{aligned} \frac{d \left[\frac{\omega^y}{P}(\phi) \right]}{d\phi} &= \frac{\partial(\omega^y/P)}{\partial\omega^y} \frac{\partial\omega^y}{\partial\phi} \\ &= \frac{(1 - \alpha^y)}{P} \frac{\partial\omega^y}{\partial\phi} > 0, \text{ since } \frac{\partial\omega^y}{\partial\phi} > 0. \end{aligned}$$

Next, under wage-smoothing equilibrium, we can use equation (13) to obtain

$$\begin{aligned} \frac{d \left[\frac{\omega^*}{P}(\phi) \right]}{d\phi} &= \frac{\partial(\omega^*/P)}{\partial\omega^y} \frac{\partial\omega^y}{\partial\phi} \\ &= \frac{\mu'(\omega^y/P)}{\mu'(\omega^*/P)} \frac{\partial(\omega^y/P)}{\partial\omega^y} \frac{\partial\omega^y}{\partial\phi} \\ &= \frac{\mu'(\omega^y/P)}{\mu'(\omega^*/P)} \frac{(1 - \alpha^y)}{P} \frac{\partial\omega^y}{\partial\phi} > 0, \text{ since } \frac{\partial\omega^y}{\partial\phi} > 0. \end{aligned}$$

Similarly, under fluctuating-wage equilibrium, we can use equation (20) to get

$$\begin{aligned}
\frac{d \left[\frac{E_\epsilon \omega_\epsilon}{P}(\phi) \right]}{d\phi} &= \frac{\partial(E_\epsilon \omega_\epsilon / P)}{\partial \omega^y} \frac{\partial \omega^y}{\partial \phi} + \frac{\partial(E_\epsilon \omega_\epsilon / P)}{\partial Q^E} \frac{\partial Q^E(\phi)}{\partial \phi} \\
&= \frac{\mu'(\omega^y / P)}{E_\epsilon \mu'(\omega_\epsilon / P)} \frac{\partial(\omega^y / P)}{\partial \omega^y} \frac{\partial \omega^y}{\partial \phi} + \frac{\partial(E_\epsilon \omega_\epsilon / P)}{\partial Q^E} \frac{\partial Q^E(\phi)}{\partial \phi} \\
&= \frac{\mu'(\omega^y / P)}{E_\epsilon \mu'(\omega_\epsilon / P)} \frac{(1 - \alpha^y)}{P} \frac{\partial \omega^y}{\partial \phi} + \frac{\partial(E_\epsilon \omega_\epsilon / P)}{\partial Q^E} \frac{\partial Q^E(\phi)}{\partial \phi} \\
&> 0, \text{ since } \frac{\partial \omega^y}{\partial \phi} > 0, \frac{\partial(E_\epsilon \omega_\epsilon / P)}{\partial Q^E} > 0 \text{ and } \frac{\partial Q^E(\phi)}{\partial \phi} > 0.
\end{aligned}$$

Furthermore, given that $\mu(\cdot)$ is strictly concave, $\frac{E_\epsilon \omega_\epsilon}{P}(\phi) > \frac{\omega^y}{P}(\phi)$ and $\frac{\omega^*}{P}(\phi) > \frac{\omega^y}{P}(\phi)$, we can conclude that

$$\begin{aligned}
\frac{d \left[\frac{\omega_G}{P}(\phi) \right]}{d\phi} &> \frac{d \left[\frac{E_\epsilon \omega_\epsilon}{P}(\phi) \right]}{d\phi} > \frac{d \left[\frac{\omega^y}{P}(\phi) \right]}{d\phi} > 0 > \frac{d \left[\frac{\omega_B}{P}(\phi) \right]}{d\phi}, \text{ and} \\
\frac{d \left[\frac{\omega^*}{P}(\phi) \right]}{d\phi} &> \frac{d \left[\frac{\omega^y}{P}(\phi) \right]}{d\phi} > 0.
\end{aligned} \tag{39}$$

Now, we are ready to look at the economy-wide real wage volatility under each type of equilibria. First, consider the fluctuating-wage equilibrium. There are three types of wages at the steady state: for those that are in the second or later periods of their X -sector employment, we have the good-state wage, ω_G and the bad-state wage, ω_B , and for those that are either employed in the Y sector or in the first period of their X -sector employment, we have Y -sector wage, ω^y . The proportion of workers that receive a particular type of wage is given below.

Sector of Employment	Employment Period	Wage	Proportion of Workers
X	2^{nd} or later	$\omega_G(\phi)$	$\rho \pi_G \left(\frac{L-n(\phi)}{L} \right)$
X	2^{nd} or later	$\omega_B(\phi)$	$\rho(1 - \pi_G) \left(\frac{L-n(\phi)}{L} \right)$
X	1^{st}	$\omega^y(\phi)$	$(1 - \rho) \left(\frac{L-n(\phi)}{L} \right)$
Y	Any	$\omega^y(\phi)$	$\frac{n(\phi)}{L}$

Let Pr_G , Pr_B , and Pr_Y represent the steady-state probability that a worker receives the good-state wage, the bad state wage and the Y -sector wage, respectively. Using the table

above, we have

$$\begin{aligned}
Pr_G(\phi) &= \rho\pi_G \left(\frac{L - n(\phi)}{L} \right) \\
Pr_B(\phi) &= \rho(1 - \pi_G) \left(\frac{L - n(\phi)}{L} \right) \\
Pr_Y(\phi) &= (1 - \rho) \left(\frac{L - n(\phi)}{L} \right) + \frac{n(\phi)}{L} \\
&= 1 - \rho \left(\frac{L - n(\phi)}{L} \right).
\end{aligned}$$

Denote $\bar{\omega}(\phi)$ as the expected nominal wage of a worker who might be working either in the X or Y sector. Then, the expected real wage of a worker is given by

$$\begin{aligned}
\frac{\bar{\omega}(\phi)}{P(\phi)} &= Pr_G(\phi) \frac{\omega_G(\phi)}{P(\phi)} + Pr_B(\phi) \frac{\omega_B(\phi)}{P(\phi)} + Pr_Y(\phi) \frac{\omega^y(\phi)}{P(\phi)}, \text{ which is} \\
&= (1 - Pr_Y(\phi)) \frac{E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} + Pr_Y(\phi) \frac{\omega^y(\phi)}{P(\phi)}.
\end{aligned} \tag{40}$$

Denote $\sigma_{\frac{\omega}{P}}^2(\phi)$ the variance of the real wage. It can be easily calculated as

$$\begin{aligned}
\sigma_{\frac{\omega}{P}}^2(\phi) &= Pr_G(\phi) \left(\frac{\omega_G(\phi) - \bar{\omega}(\phi)}{P(\phi)} \right)^2 + Pr_B(\phi) \left(\frac{\omega_B(\phi) - \bar{\omega}(\phi)}{P(\phi)} \right)^2 \\
&\quad + Pr_Y(\phi) \left(\frac{\omega^y(\phi) - \bar{\omega}(\phi)}{P(\phi)} \right)^2, \text{ or} \\
\sigma_{\frac{\omega}{P}}^2(\phi) &= Pr_G(\phi) \left(\frac{[\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)] + [E_\epsilon \omega_\epsilon(\phi) - \bar{\omega}(\phi)]}{P(\phi)} \right)^2 \\
&\quad + Pr_B(\phi) \left(\frac{[\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)] + [E_\epsilon \omega_\epsilon(\phi) - \bar{\omega}(\phi)]}{P(\phi)} \right)^2 \\
&\quad + Pr_Y(\phi) \left(\frac{\omega^y(\phi) - \bar{\omega}(\phi)}{P(\phi)} \right)^2.
\end{aligned}$$

Using equation (40), we can replace $[\omega^y(\phi) - \bar{\omega}(\phi)]$ with $(1 - Pr_Y(\phi)) [\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)]$ and

also $[E_\epsilon \omega_\epsilon(\phi) - \bar{\omega}(\phi)]$ with $Pr_Y(\phi) [E_\epsilon \omega_\epsilon(\phi) - \omega^y(\phi)]$ to obtain

$$\begin{aligned} \sigma_{\frac{\omega}{P}}^2(\phi) &= Pr_G(\phi) \left[\left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2 + [Pr_Y(\phi)]^2 \left(\frac{E_\epsilon \omega_\epsilon(\phi) - \omega^y(\phi)}{P(\phi)} \right)^2 \right] \\ &\quad + 2Pr_Y(\phi) \left(\frac{E_\epsilon \omega_\epsilon(\phi) - \omega^y(\phi)}{P(\phi)} \right) \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right) \\ &\quad + Pr_B(\phi) \left[\left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2 + [Pr_Y(\phi)]^2 \left(\frac{E_\epsilon \omega_\epsilon(\phi) - \omega^y(\phi)}{P(\phi)} \right)^2 \right] \\ &\quad + 2Pr_Y(\phi) \left(\frac{E_\epsilon \omega_\epsilon(\phi) - \omega^y(\phi)}{P(\phi)} \right) \left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right) \\ &\quad + Pr_Y(\phi) [1 - Pr_Y(\phi)]^2 \left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2. \end{aligned}$$

The above equation can be rearranged as

$$\begin{aligned} \sigma_{\frac{\omega}{P}}^2(\phi) &= Pr_G(\phi) \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2 + Pr_B(\phi) \left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2 \\ &\quad + [Pr_G(\phi) + Pr_B(\phi)] [Pr_Y(\phi)]^2 \left(\frac{E_\epsilon \omega_\epsilon(\phi) - \omega^y(\phi)}{P(\phi)} \right)^2 \\ &\quad + 2Pr_Y(\phi) \left(\frac{E_\epsilon \omega_\epsilon(\phi) - \omega^y(\phi)}{P(\phi)} \right) \left[Pr_G(\phi) \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right) \right. \\ &\quad \left. + Pr_B(\phi) \left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right) \right] \\ &\quad + Pr_Y(\phi) [1 - Pr_Y(\phi)]^2 \left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2. \end{aligned}$$

Notice that

$$\begin{aligned} Pr_G(\phi) \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right) + Pr_B(\phi) \left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right) &= 0, \text{ and} \\ Pr_G(\phi) + Pr_B(\phi) &= 1 - Pr_Y(\phi). \end{aligned}$$

Using these two, we obtain

$$\begin{aligned} \sigma_{\frac{\omega}{P}}^2(\phi) &= [1 - Pr_Y(\phi)] \left[\pi_G \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2 + (1 - \pi_G) \left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2 \right] \\ &\quad + Pr_Y(\phi) [1 - Pr_Y(\phi)] \left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2, \end{aligned} \quad (41)$$

where on the right-hand side, the first line represents the within-group variance of those earning fluctuating wage in the X sector weighted by its steady-state probability and the second line represents between-group wage variance. Therefore, the variance consists of sum of within group variance and between group variance.

Differentiating equation (41) with respect to ϕ , we obtain

$$d\sigma_{\frac{\omega}{P}}^2(\phi) = \left\{ \begin{aligned} & [1 - Pr_Y(\phi)] d \left[\pi_G \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2 + (1 - \pi_G) \left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2 \right] \\ & + Pr_Y(\phi) [1 - Pr_Y(\phi)] d \left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2 \\ & \left\{ + \left[\pi_G \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2 + (1 - \pi_G) \left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2 \right] d [1 - Pr_Y(\phi)] \right. \\ & \left. + \left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P(\phi)} \right)^2 d Pr_Y(\phi) [1 - Pr_Y(\phi)] \right\} \end{aligned} \right. \quad (42)$$

where the terms in the first brace represent the ‘price’ effect, which measures the change in wage volatility while holding industrial composition intact, i.e., keeping the proportion of workers in each sector constant. The terms in the second brace represent the ‘compositional’ effect, which measures the change in wage volatility in response to changes in sectoral compositions only, i.e., the movement of workers between sectors with different degrees of wage volatility. We can rewrite the ‘price’ effect as

$$= 2 [1 - Pr_Y(\phi)] \left\{ \begin{aligned} & \pi_G \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)^2 \left[\frac{d \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)}{\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P}} \right] \\ & + (1 - \pi_G) \left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)^2 \left[\frac{d \left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)}{\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P}} \right] \\ & + Pr_Y(\phi) \left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)^2 \left[\frac{d \left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)}{\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P}} \right] \end{aligned} \right\} > 0 \text{ for } d\phi > 0.$$

Using conditions given in (39), we can see that the ‘price’ effect is always positive. We can also rewrite the ‘compositional’ effect as

$$= \rho \frac{n}{L} \left[-\frac{dn(\phi)}{n} \right] \left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)^2 \left\{ \begin{aligned} & \frac{\pi_G \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)^2 + (1 - \pi_G) \left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)^2}{\left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)^2} \\ & + 1 - 2\rho \left(\frac{L-n(\phi)}{L} \right) \end{aligned} \right\}.$$

Notice that we have $1 - 2\rho \left(\frac{L-n(\phi)}{L} \right) > -1$, since $\rho \left(\frac{L-n(\phi)}{L} \right) = [1 - Pr_Y(\phi)] \in (0, 1)$. In addition, we know that $dn(\phi) < 0$ for $d\phi > 0$. Therefore, a sufficient condition for the compositional effect to be non-negative is

$$\frac{\pi_G \left(\frac{\omega_G(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)^2 + (1 - \pi_G) \left(\frac{\omega_B(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)^2}{\left(\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P} \right)^2} \geq 1, \text{ or}$$

$$\pi_G (1 - \pi_G) \left(\frac{\frac{\omega_G(\phi) - \omega_B(\phi)}{P}}{\frac{\omega^y(\phi) - E_\epsilon \omega_\epsilon(\phi)}{P}} \right)^2 \geq 1.$$

If this condition is satisfied, then the compositional effect is non-negative and economy-wide wage volatility increases as a result of an improvement in search efficiency.

After some algebra, we can rewrite equation (42) as

$$d\sigma_{\frac{\omega}{P}}^2(\phi) = \begin{cases} 2Pr_Y(\phi) [1 - Pr_Y(\phi)] \left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon\omega_\epsilon}{P}(\phi)\right)^2 \left[-\frac{dn(\phi)}{n}\right] \\ \left[\gamma_G \varepsilon(\omega_G - E_\epsilon\omega_\epsilon), n - \gamma_B \varepsilon(\omega_B - E_\epsilon\omega_\epsilon), n + \varepsilon(\omega^y - E_\epsilon\omega_\epsilon), n \right] \\ * \left[+\frac{1}{2} \frac{\frac{n(\phi)}{L}}{1 - \frac{n(\phi)}{L}} \left(\gamma_G + \gamma_B + 1 - \frac{1 - Pr_Y(\phi)}{Pr_Y(\phi)} \right) \right] \end{cases}, \text{ where} \quad (43)$$

$$\varepsilon(\omega_\epsilon - E_\epsilon\omega_\epsilon), n = \frac{\left[\frac{d\left(\frac{\omega_\epsilon}{P}(\phi) - \frac{E_\epsilon\omega_\epsilon}{P}(\phi)\right)}{\frac{\omega_\epsilon}{P}(\phi) - \frac{E_\epsilon\omega_\epsilon}{P}(\phi)} \right]}{\left[-\frac{dn(\phi)}{n} \right]} > 0, \quad \varepsilon(\omega^y - E_\epsilon\omega_\epsilon), n = \frac{\left[\frac{d\left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon\omega_\epsilon}{P}(\phi)\right)}{\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon\omega_\epsilon}{P}(\phi)} \right]}{\left[-\frac{dn(\phi)}{n} \right]} > 0,$$

$$\gamma_G = \frac{\pi_G}{Pr_Y(\phi)} \frac{\left(\frac{\omega_G}{P}(\phi) - \frac{E_\epsilon\omega_\epsilon}{P}(\phi)\right)^2}{\left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon\omega_\epsilon}{P}(\phi)\right)^2} > 0, \quad \gamma_B = \frac{(1 - \pi_G)}{Pr_Y(\phi)} \frac{\left(\frac{\omega_B}{P}(\phi) - \frac{E_\epsilon\omega_\epsilon}{P}(\phi)\right)^2}{\left(\frac{\omega^y}{P}(\phi) - \frac{E_\epsilon\omega_\epsilon}{P}(\phi)\right)^2} > 0.$$

Here, ε 's represent employment elasticity of wage differences, $(\gamma_G + \gamma_B)$ is the ratio of within group variance (weighted by its steady-state probability) to between group variance, and γ_G and γ_B are the contribution of within group good-state and bad-state wage variance to this ratio. From equation (43), we can see that the economy-wide real wage volatility increases as a result of an improvement in search effectiveness ($d\phi > 0$) iff

$$\gamma_G \varepsilon(\omega_G - E_\epsilon\omega_\epsilon), n - \gamma_B \varepsilon(\omega_B - E_\epsilon\omega_\epsilon), n + \varepsilon(\omega^y - E_\epsilon\omega_\epsilon), n > -\frac{1}{2} \frac{\frac{n(\phi)}{L}}{1 - \frac{n(\phi)}{L}} \left(\gamma_G + \gamma_B + 1 - \frac{1 - Pr_Y(\phi)}{Pr_Y(\phi)} \right).$$

Next, consider the wage-smoothing equilibrium. There are two types of wages at the steady state: for those that are in the second or later periods of their X -sector employment, we have the efficiency wage, ω^* , and for those that are either employed in the Y sector or in the first period of their X -sector employment, we have Y -sector wage, ω^y . The proportion of workers that receive a particular type of wage is given below.

Sector of Employment	Employment Period	Wage	Proportion of Workers
X	2 nd or later	$\omega^*(\phi)$	$\rho \left(\frac{L - n(\phi)}{L} \right)$
X	1 st	$\omega^y(\phi)$	$(1 - \rho) \left(\frac{L - n(\phi)}{L} \right)$
Y	Any	$\omega^y(\phi)$	$\frac{n(\phi)}{L}$

Using the table above, we have $Pr_Y(\phi) = 1 - \rho \left(\frac{L-n(\phi)}{L} \right)$ as before and the expected real wage of a worker who might be working either in the X or Y sector is given by

$$\frac{\bar{\omega}(\phi)}{P} = (1 - Pr_Y(\phi)) \frac{\omega^*(\phi)}{P(\phi)} + Pr_Y(\phi) \frac{\omega^y(\phi)}{P(\phi)}. \quad (44)$$

The variance of the real wage can be easily calculated as

$$\sigma_{\frac{\omega}{P}}^2(\phi) = [1 - Pr_Y(\phi)] \left(\frac{\omega^*(\phi) - \bar{\omega}(\phi)}{P(\phi)} \right)^2 + Pr_Y(\phi) \left(\frac{\omega^y(\phi) - \bar{\omega}(\phi)}{P(\phi)} \right)^2.$$

Using equation (44), we can replace $[\omega^y(\phi) - \bar{\omega}(\phi)]$ with $(1 - Pr_Y(\phi)) [\omega^y(\phi) - \omega^*(\phi)]$ and also $[\omega^*(\phi) - \bar{\omega}(\phi)]$ with $Pr_Y(\phi) [\omega^*(\phi) - \omega^y(\phi)]$ to obtain

$$\sigma_{\frac{\omega}{P}}^2(\phi) = Pr_Y(\phi) [1 - Pr_Y(\phi)] \left(\frac{\omega^*(\phi) - \omega^y(\phi)}{P(\phi)} \right)^2. \quad (45)$$

Differentiating equation (45) with respect to ϕ , we obtain

$$\begin{aligned} d\sigma_{\frac{\omega}{P}}^2(\phi) &= \left\{ Pr_Y(\phi) [1 - Pr_Y(\phi)] d \left(\frac{\omega^*(\phi) - \omega^y(\phi)}{P(\phi)} \right)^2 \right. \\ &\quad \left. + \left(\frac{\omega^*(\phi) - \omega^y(\phi)}{P(\phi)} \right)^2 dPr_Y(\phi) [1 - Pr_Y(\phi)] \right\} \end{aligned} \quad (46)$$

As before, the terms in the first brace represent the ‘price’ effect and the terms in the second brace represent the ‘compositional’ effect. We can rewrite the ‘price’ effect as

$$= 2Pr_Y(\phi) [1 - Pr_Y(\phi)] \left(\frac{\omega^*}{P}(\phi) - \frac{\omega^y}{P}(\phi) \right)^2 \left[\frac{d \left(\frac{\omega^*}{P}(\phi) - \frac{\omega^y}{P}(\phi) \right)}{\frac{\omega^*}{P}(\phi) - \frac{\omega^y}{P}(\phi)} \right] > 0, \text{ for } d\phi > 0.$$

Again using condition given in (39), we can see that the ‘price’ effect is always positive. We can also rewrite the ‘compositional’ effect as

$$= \rho \frac{n}{L} \left[-\frac{dn(\phi)}{n} \right] \left(\frac{\omega^*}{P}(\phi) - \frac{\omega^y}{P}(\phi) \right)^2 \left[1 - 2\rho \left(\frac{L-n(\phi)}{L} \right) \right].$$

Notice that we have $1 - 2\rho \left(\frac{L-n(\phi)}{L} \right) = 1 - 2[1 - Pr_Y(\phi)]$. As before, we also know that $dn(\phi) < 0$, for $d\phi > 0$. Thus, a sufficient condition for the compositional effect to be non-negative is

$$\begin{aligned} 1 - 2[1 - Pr_Y(\phi)] &\geq 0, \text{ or} \\ Pr_Y(\phi) &\geq \frac{1}{2}. \end{aligned}$$

If this condition is satisfied, then the compositional effect is non-negative and economy-wide real wage volatility increases as a result of an improvement in search efficiency.

After some algebra, we can rewrite equation (46) as

$$d\sigma_{\frac{\omega}{P}}^2(\phi) = \begin{cases} 2Pr_Y(\phi) [1 - Pr_Y(\phi)] \left(\frac{\omega^y}{P}(\phi) - \frac{\omega^*}{P}(\phi)\right)^2 \left[-\frac{dn(\phi)}{n}\right] \\ * \left[\varepsilon_{(\omega^y - \omega^*), n} + \frac{1}{2} \frac{\frac{n(\phi)}{L}}{1 - \frac{n(\phi)}{L}} \left(1 - \frac{1 - Pr_Y(\phi)}{Pr_Y(\phi)}\right) \right], \end{cases} \quad (47)$$

where $\varepsilon_{(\omega^y - \omega^*), n} = \frac{\left[\frac{d\left(\frac{\omega^y}{P}(\phi) - \frac{\omega^*}{P}(\phi)\right)}{\frac{\omega^y}{P}(\phi) - \frac{\omega^*}{P}(\phi)}\right]}{\left[-\frac{dn(\phi)}{n}\right]} > 0$. From equation (47), we can see that the economy-wide real wage volatility increases as a result of an improvement in search efficiency ($d\phi > 0$) iff

$$\varepsilon_{(\omega^y - \omega^*), n} > -\frac{1}{2} \frac{\frac{n(\phi)}{L}}{1 - \frac{n(\phi)}{L}} \left(1 - \frac{1 - Pr_Y(\phi)}{Pr_Y(\phi)}\right). \blacksquare$$

References

- [1] Andrews, E.L., 1996. AT&T's plan for cuts: be gentle but stay firm. *The New York Times*, February 13.
- [2] Autor, D.H., 2001a. Why do temporary help firms provide free general skills training? *Quarterly Journal of Economics* 116, pp. 1409-1448.
- [3] Autor, D.H., 2001b. Wiring the labor market. *Journal of Economic Perspectives* 15, pp. 25-40.
- [4] Autor, D.H., F. Levy and R. Murnane, 1999. Skills training in the temporary help sector: employer motivations and worker impacts. A report to the Department of Labor Education and Training Administration, Washington, DC.
- [5] Autor, D.H., L.F. Katz and M.S. Kearney, 2008. Trends in U.S. wage inequality: revising the revisionists. *Review of Economics and Statistics* 90, pp. 300-323.

- [6] Baker, M. and G. Solon, 2003. Earnings dynamics and inequality among Canadian men, 1967-1992: evidence from longitudinal income tax records. *Journal of Labor Economics* 21, pp. 289-322.
- [7] Beaudry, P. and J. DiNardo, 1991. The effects of implicit contracts on the behavior of wages over the business cycle. *Journal of Political Economy* 99, pp. 665-688.
- [8] Benner, C., 2002. *Work in the New Economy: Flexible Labor Markets in Silicon Valley*. Oxford: Blackwell Publishers.
- [9] Carnoy, M. and M. Castells, 1997. Sustainable flexibility: a prospective study on work, family and society in the information age. OECD Working Papers 5, No. 29.
- [10] Castells M., 1996. *The Rise of the Network Society*. Cambridge, MA: Blackwell Publishers.
- [11] Dickens, R., 2000. The evolution of individual male earnings in Great Britain: 1975-1995. *Economic Journal* 110, pp. 27-49.
- [12] Farber, H., 2009. Job loss and the decline in job security in the United States. Princeton University Industrial Relations Section Working Paper No. 520.
- [13] Finlay, W. and J.E. Coverdill, 2002. *Headhunters: Matchmaking in the Labor Market*. Ithaca, NY: Cornell University Press.
- [14] Gosselin, P., 2008. *High Wire: The Precarious Financial Lives of American Families*. Basic Books.
- [15] Gottschalk, P., R. Moffitt, L.F. Katz and W.T. Dickens, 1994. The growth of earnings instability in the U.S. labor market." *Brookings Papers on Economic Activity* 2, pp. 217-272.
- [16] Hacker, J.S. and E. Jacobs, 2008. The rising instability of American family incomes, 1969-2004: evidence from the panel study of income dynamics. EPI Briefing Paper 213 (May), Economic Policy Institute.

- [17] Harris, M. and B. Holmström, 1982. A theory of wage dynamics. *Review of Economic Studies* 49, pp. 315-333.
- [18] Holmström, B., 1983. Equilibrium long-term labour contracts. *Quarterly Journal of Economics* (Supplement) 98, pp. 23-54.
- [19] Kambourov, G. and I. Manovskii, 2009. Occupational mobility and wage inequality. *Review of Economic Studies* 76, pp. 731-759.
- [20] Karabay, B. and J. McLaren, 2010. Trade, offshoring, and the invisible handshake. *Journal of International Economics* 82, pp. 26-34.
- [21] Katz, L.F. and A.B. Krueger, 1999. The high-pressure U.S. labor market of the 1990s. *Brookings Papers on Economic Activity* 1, pp. 1-65.
- [22] Kazis, R., 1998. New Labor Market Intermediaries: What's driving them? Where are they headed? Task Force Working Paper No. 03, MIT Sloan School of Management, Cambridge, MA.
- [23] Korzeniewicz, M. and G. Gereffi, 1994. *Commodity Chains and Global Capitalism*. Westport, CT: Praeger.
- [24] Kranton, R.E., 1996. Reciprocal exchange: a self-sustaining system. *American Economic Review* 86, pp. 830-851.
- [25] Kroft, K. and D. Pope, *forthcoming*. Does Online Search Crowd-out Traditional Search and Improve Matching Efficiency? Evidence from Craigslist. *Journal of Labor Economics*.
- [26] Kuhn, P., *forthcoming*. The internet as a labor market matchmaker: how effective are the new electronic methods of workers recruitment and job search? *IZA World of Labor*.
- [27] Kuhn, P. and H. Mansour, *forthcoming*. Is internet job search still ineffective? *Economic Journal*.

- [28] Kuhn, P. and M. Skuterud, 2004. Internet job search and unemployment durations. *American Economic Review* 94, pp. 218-232.
- [29] Lazear, E.P., 1979. Why is there mandatory retirement? *Journal of Political Economy* 87, pp. 1261-1284.
- [30] Lemieux, T., 2006. Increased residual wage inequality: composition effects, noisy data, or rising demand for skill. *American Economic Review* 96, pp. 461-498.
- [31] Malcomson, J.M., 1999. Individual employment contracts. Chapter 35 of O. Ashenfelter and D. Card, eds., *The Handbook of Labor Economics*, Vol. 3, Amsterdam: Elsevier, pp. 2291-2372.
- [32] Matouschek, N. and P. Ramezzana, 2007. The role of exclusive contracts in facilitating market transactions. *Journal of Industrial Economics* 55, pp. 197-222.
- [33] McDonald, J.T. and C. Worswick, 1999. Wages, implicit contracts and the business cycle: evidence from Canadian micro data. *Journal of Political Economy* 107, pp. 884-892.
- [34] McLaren, J., 2003. Trade and market thickness: effects on organizations. *Journal of the European Economic Association* 1, pp. 328-336.
- [35] McLaren, J. and A. Newman, 2004. Globalization and insecurity. Mimeo: University of Virginia.
- [36] Ramey, G. and J. Watson, 2001. Bilateral trade and opportunism in a matching market. *B.E. Journal of Theoretical Economics (Contributions)* 1(1), Article 3.
- [37] Sassen, S., 1996. *Losing Control? Sovereignty in an Age of Globalization*. New York: Columbia University Press.
- [38] Schettkat, R. and L. Yocarini. The shift to services employment: a review of the literature. *Structural Change and Economic Dynamics* 17, pp. 127-147.

- [39] Segal, L. and D. Sullivan, 1997. Temporary services employment durations: evidence from state UI data. Working Paper Series, Macroeconomic Issues WP-97-23, Federal Reserve Bank of Chicago.
- [40] Shleifer, A. and L.H. Summers, 1988. Breach of trust in hostile takeovers. In Alan J. Auerbach, ed., *Corporate Takeovers: Causes and Consequences*, Chicago, IL: The University of Chicago Press, pp. 33-56.
- [41] Stevenson, B., 2006. The impact of the internet on worker flows. Mimeo: The Wharton School, University of Pennsylvania (December).
- [42] Thomas, J. and T. Worrall, 1988. Self-enforcing wage contracts. *Review of Economic Studies*, 55, pp. 541-554.
- [43] Uren, L., 2008. Inequality, volatility and labour market efficiency. *B.E. Journal of Macroeconomics (Topics)* 8(1), Article 17.

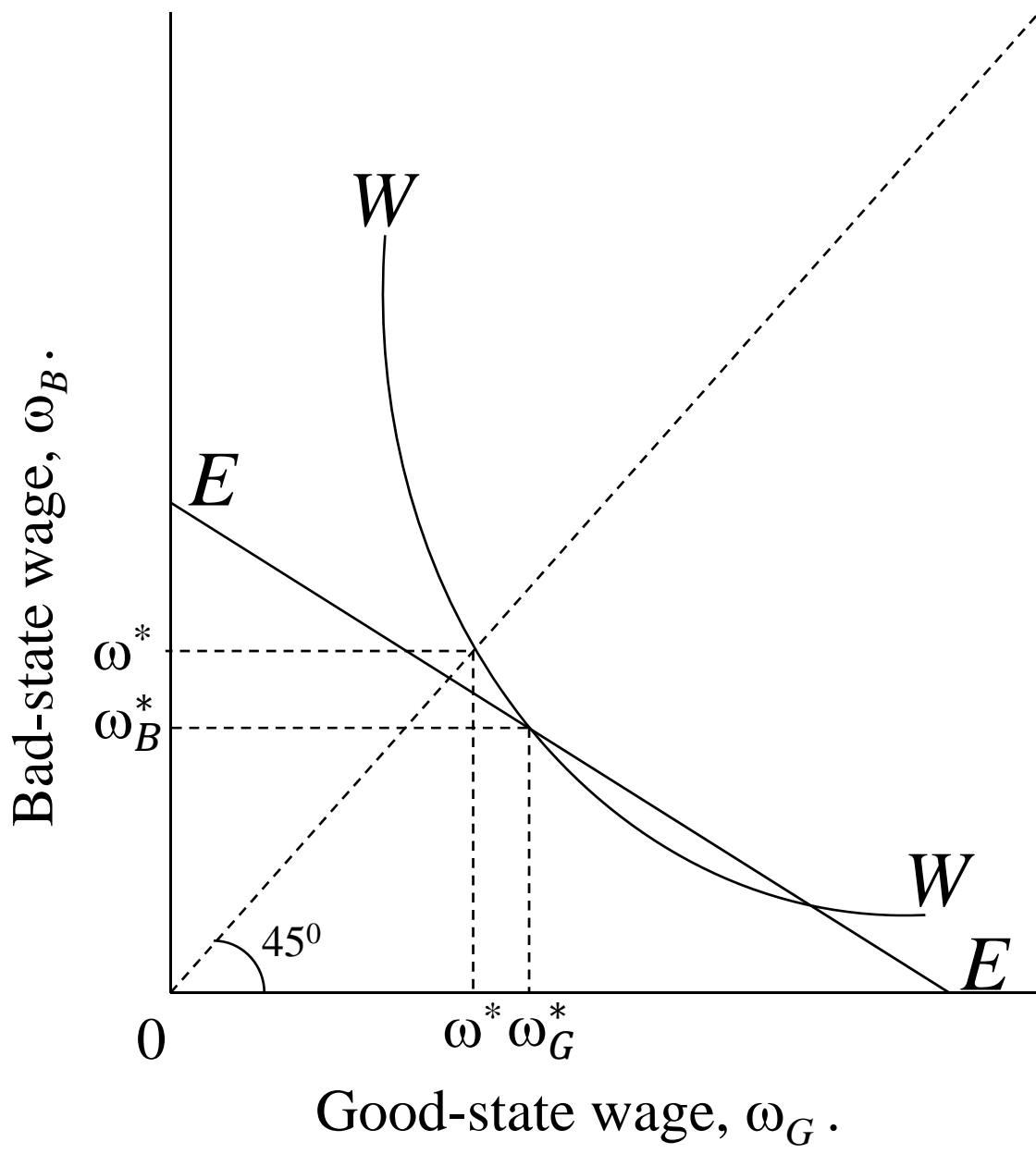


Figure 1. Fluctuating-wage equilibrium.

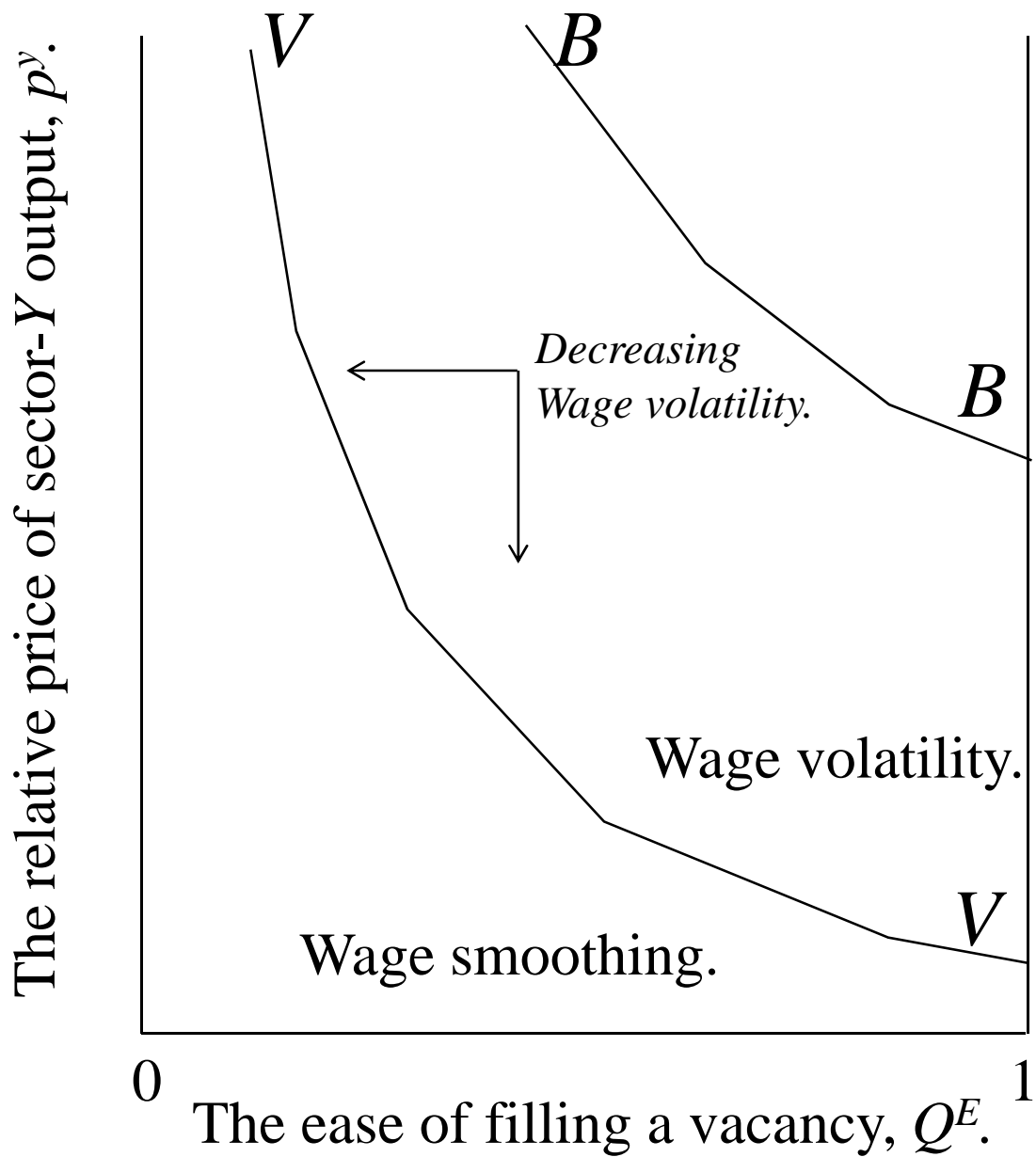


Figure 2. Type of Wage Contract and Comparative Statics.

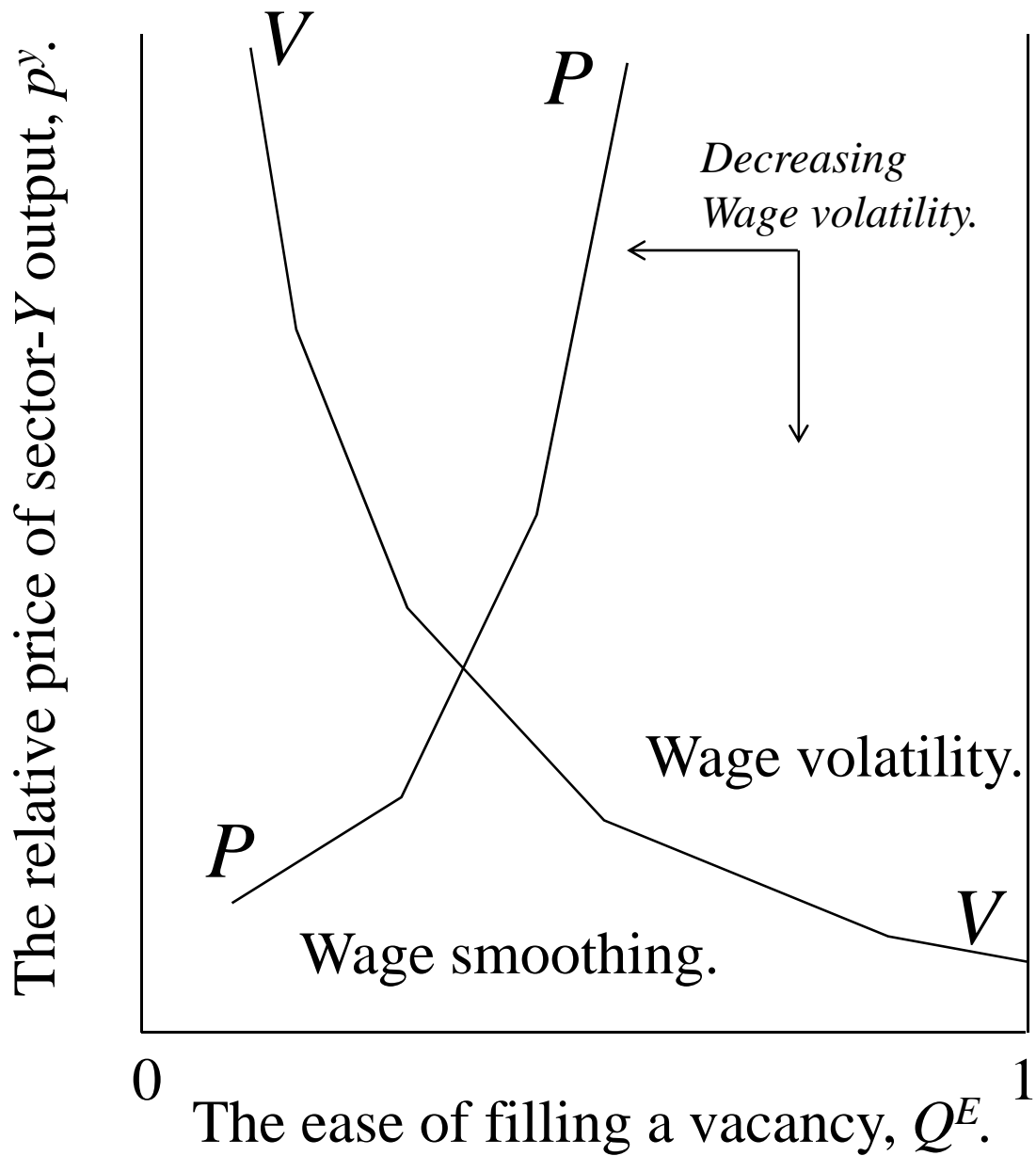


Figure 3. The effects of an improvement in search effectiveness.