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Impatient in Experiments, but Patient in Simulations: A Challenge to a Neoclassical Model

Emin Gahramanov and Xueli Tang

Impatient in Experiments, but Patient in Simulations: A Challenge to a Neoclassical Model.*

Emin Gahramanov† Xueli Tang

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Abstract

Despite ample empirical evidence on the prevalence of high discount rates among people, applied, quantitative-theoretical macro studies with exponential discounting often assume low positive, or even negative discount rate values. Relying on recent advances from the numerical optimal control branch of mathematics, we solve a neoclassical, continuous time model of endogenous consumption/saving and labor supply, and show that even if an agent has a moderately high discount rate, his labour supply and consumption behavior will be highly counterfactual. We provide a remedy to such counterfactual findings by augmenting a standard utility function based on recent evidences from the leisure sciences, while maintaining a rational choice approach of neoclassical economics.

JEL Classification: D91, C02, C61, J22, J26

Key Words: Bounded control; Numerical Optimal Control; Life-cycle Consumption and Labor-Leisure

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†The authors are in the School of Accounting, Economics and Finance at Deakin University. Send correspondence to Gahramanov at: eming@deakin.edu.au. Tel.: +61 3 925 17467

1 Introduction

Choice data and evidence often suggests double-digit discount rates for a large spectrum of individuals. For example, Loewenstein and Thaler (1989) provides evidence of extremely high discount rates among many high school dropouts, sun lovers, and homeowners. Coller and Williams (1999) recognize that subjects may substitute field alternatives to lab incentives. Thus to better evaluate subject responses, the authors provide more information in the laboratory on alternative payment schemes and market interest rates. The result is that median annual individual discount rates fall within a 17.5-20% range. Warner and Pleeter (2001) study real choices made by 65,000 separatees (due to a military downsizing program) over large sums of money. The authors find personal discount rates to be between about 17.5-19.8 percent. Harrison et al. (2002) use survey questions with real monetary rewards in Denmark, and estimate the average discount rate to be about 28 percent.[1]

However, despite these solid evidences, quantitative-theoretical macro studies assume a low (or very moderate) degree of impatience. Specifically, high frequency macro models of intertemporal choices assume exponential discounting with the discount rate of around 3.5% per annum, while some studies even assume slightly negative discount rates. In this study, we will show that a traditional neoclassical model of intertemporal choice is seriously challenged if one assumes an agent with a high discount rate. We will also suggest a simple remedy to those challenges.

Let us first ask the following question: if preferences are consistent, but many people are highly impatient, why do macro studies that analyze important, policy relevant issues, assume a low degree of impatience? One reason is that with high discount rates it is difficult to obtain a realistic level of aggregate capital-output ratio. Indeed, a standard, neoclassical overlapping-generations model of intertemporal consumption/saving choice can easily reconcile the observed capital-output ratio if people discount future utility at around 2-6% per annum, as argued in Feigenbaum, Gahramanov, and Tang (2013). Yet the authors argue that with double-digit discount rates, the model stumbles. The authors further suggest to completely abandon the neoclassical model of an individually rational choice in favour of an *optimal irrational behavior* framework (see Feigenbaum and Caliendo (2010), Feigenbaum, Caliendo, and Gahramanov (2011)), as this allows reconciliation between the high discount rate with high capital accumulation. Intuitively, a rational household allocates less consumption to old age if the discount rate increases, therefore its saving declines with the discount rate. This, however, does not produce the highest utility among feasible market equilibria. If the discount rate is very high, saving late in life will not decrease utility that much while permitting a higher lifetime consumption in general equilibrium. Consequently, Feigenbaum, Gahramanov, and Tang find that for some parameterization of the model, the capital-output ratio can actually *increase* with the discount rate in a (restricted) optimally irrational environment.[2]

In this study, we focus on both the intertemporal consumption and the labor supply choice in a

[1] Morrison and Oxoby (2013) have recently conducted a laboratory experiment involving intertemporal choices made by undergraduate students at the University of Calgary, and tested for the endowment effect which points to a disincentive to save. Although no participants revealed a switching behavior, accounting for the endowment effect leads to significantly higher discount rates compared to when individuals treat monetary resources as windfall gains in the laboratory.

[2] Gourinchas and Parker (2002) and Feigenbaum (2008) showed that, with sufficient risk, precautionary saving can bring data on the discount rate and optimal life-cycle consumption path to agreement. However, Feigenbaum and Li (2012) argue that uncertainty and heterogeneity have been perplexed in earlier studies. More realistic estimates of income uncertainty produce much smaller estimates of idiosyncratic risk.

neoclassical benchmark model similar to that in Heckman (1974). Heckman suggested that bundles of consumption and leisure should jointly be determined over the lifetime. We show that a high discount rate presents yet another serious challenge to a standard, totally frictionless, intertemporal neoclassical model of consumption-saving/labour-leisure choice (CS/LL model, hereafter). Namely, if the discount rate is even at moderate double-digit levels, the optimal path of leisure often reveals *no* entrance to the job market for many years in early adulthood, and then rapidly decaying leisure consumption in later years, resulting in an agent working unrealistically intensely into his late 90s. Another interesting aspect of the CS/LL model is that if the discount rate was low (as often assumed in many studies), then perturbing other parameters that enter the standard utility function, may allow generation of a realistic life-cycle labor supply path. Yet when the discount rate is high enough, such parameter perturbation does not help much. Thus, there is a need to revisit the model if there is a strong reason to suspect that a non-trivial fraction of the population may have a high discount rate as misestimating the latter can misguide the welfare analysis of various policy reforms.

How do we propose to modify the standard CS/LL model to tackle the issue of the high discount rate mentioned above? Let us start by recalling a standard utility function used in various versions of the CS/LL model. As in Ladrón-de-Guevara et al. (1995) or Bütler (2001), assume leisure and consumption are substitutes in the instantaneous utility function, where the elasticity of substitution between consumption and leisure is one.[3] In addition, we recall the marginal utility of consumption when the latter is *zero* at *any age* equals *infinity*. This restriction on the utility function effectively prevents an *unrealistic* zero consumption at any age. We then ask why we can't reasonably conjecture the marginal utility of leisure at *old age* at a relatively high level in order to get *more realistic* labor supply and/or consumption paths? Even if one assumes that disparity between physical strength and stamina on the one hand, and experience and skills accumulated over the years on the other, effectively levels-off disparities in job productivity among people of very different ages, how reasonable is it then to assume that a 25-year old man, working, say, 12 hours a day, would suffer the same disutility of work as a 90 year old man, working the same number of hours a day on a full-time job with strict commitments and deadlines?

Recent findings may shed some light on the questions we have just asked. Kopecky (2011) mentions that the majority of older workers may find it difficult to work the same way as they used to because of job-related mental stress being more pronounced as one ages. Leisure scientists Liechty and Genoe (2013) cite evidence, that taking part in meaningful leisurely activities help older workers to improve cognitive, psychological and emotional well-being. The authors interviewed elderly focus group members who revealed the feeling of obligations and work pressures accompanied typical formal work they used to do. Liechty and Genoe state on p. 450:

"*... leisure contributes to the well-being of retirement-age men. Perceptions of leisure and its benefits among the participants led to engagement in activities aimed at helping to adjust to age related changes, including both aging bodies, and social changes that occur as one leaves the work place after many years of structure and routine.*"

Even from one's personal experience, many people may agree that as one ages, and realizes that the end is near, there is an increased sense of spending the rest of life stress-free, and engaging in meaningful

[3] As Kydland (1995, p. 134) argues, the empirical realism of such a utility function is that hours in market activity are quite insensitive to historically observed changes in real wages.

leisurely activities with family and friends, looking after grandchildren, and simply enjoying the time left. What we highlight is that even though people can display a *high* time discount rate on future monetary payoffs (often observed in survey data and experimental studies), this does not imply that such impatient people do not have very good reasons to value leisure at old age differently from what they used to whilst young, controlling for real wages and on-the-job productivity levels. We show that augmenting a standard neoclassical model with a time-varying term to capture an increase in the marginal utility of leisure as one ages, is a viable remedy to unrealistic labor supply and consumption paths even for highly impatient economic agents. We observe that even a perfectly rational (yet impatient) neoclassical agent, smoothing consumption and leisure intertemporally, yet having augmented preferences, can still display a moderately hump-shaped consumption path, and can also voluntarily sustain a very abrupt drop in consumption at retirement. It is not uncommon to believe that empirically observed discrete drops in aggregate consumption data are due to the prevalence of irrational, nearly hand-to-mouth Keynesian consumers, but Huang and Caliendo (2011) argue that such discrete drops in aggregate consumption at retirement are anticipated. Our result confirms the authors' argument. In addition, it is often the case that governments and applied studies call for "forcing" people to retire later, and this call becomes even louder every year in the wake of demographic and social security challenges. We need to be mindful that pushing people who become averse to work when they are old can be highly welfare-detrimental, and may also prompt people to distort their intertemporal decision making. We further hope to provoke empirical and experimental investigations of our assumptions behind people's intertemporal preferences over leisure (and not just over monetary payoffs), as proper specifications of preferences may help avoid potentially erroneous predictions in applied welfare analyses.[4]

It should be noted we are not the first to point out the idea that disutility from labor can be time-dependent, yet this assumption has often been disregarded in the literature. A notable exception is Bommier et al. (2011), who model a zero interest, zero discount rate environment with extensive margin of labor supply, and also suggest that labor disutility at any age is a strictly increasing function of age, that enables the assumption individuals optimally choose their activity in early periods of their life. The authors assume preferences that are separable across periods and also between consumption and work, and make an important theoretical contribution by showing the socially optimal design of the pension system would imply longer-lived individuals retiring later and consuming smaller amounts per period. We focus on both margins of the labor supply, and make a quantitative contribution by investigating how significant a dynamic disutility from labor should be to correct for a highly unconventional pattern of labor and consumption paths when the discount rate is high.

We are able to resolve the above-mentioned "awkward" predictions of the benchmark, the high discount rate neoclassical model, by abstaining from the introduction of any frictions, or any form

[4]In the tradition of the large body of related literature, this study focuses on time consistent preferences, and abstracts from other forms, like hyperbolic discounting. The latter has also been a central focus in the literature, however Xia (2011) was able to explain the time preference reversal feature of hyperbolic discounting (and other common anomalies) by assuming the magnitudes of future payoffs are associated with uncertainty, and are increasing with time into the future. It follows that risk averse individuals' expected utility for future payoffs decline with the time delay. In the context of our study with no uncertainty but hyperbolic preferences, it might be that our results would become even more pronounced. As Findley and Caliendo (2013), who recently studied the effect of naïve hyperbolic preferences on the extensive margin of the labor supply (endogenous timing of retirement) found, individuals plan to retire earlier than they really do. Thus, in the context of our study with hyperbolic preferences, we may get an individual consuming even less leisure at old age compared to what they originally intended at the start of their life. We leave this issue for future research.

of irrationality in a traditional, continuous time CS/LL model. We need to say a few words about some peculiarities, surrounding the usage of the CS/LL model in economics.[5] Although such a model has been long analyzed by Heckman (1974), and then revisited by Bütler (2001), Gahramanov and Tang (2013; 2014) argued that the solutions presented in these papers were not general as the authors' primary focus was on an interior solution. Technically, there is a time endowment constraint on leisure consumption in a corresponding bounded control problem. There is much anecdotal evidence where some researchers, even outside the economics discipline, ignore a binding constraint for the sake of simplicity, and truncate an unconstrained control solution whenever it binds.[6] Well-known mathematical economists have explicitly warned against such practice (Kamien and Schwartz (1981)), but so far this issue has received little attention. Further studies on CS/LL models would typically consider a binding leisure constraint, yet would not fully capitalize on the implications of the leisure constraint analysis to obtain actual solutions of lifetime consumption and leisure paths. Gahramanov and Tang (2013) solved the CS/LL model analytically, and provided complete and explicit solutions to cases when the labour supply path did not bind (the individual always works), or binded only once (the individual initially works for part of his lifetime, then retires permanently thereafter). As a CS/LL problem is a typical problem of the optimal control branch of mathematics, to be able to analyze a wider spectrum of possible solutions, in this paper we follow the recommendations of the optimal control scientists (see, e.g., Gregory and Lin (1992), and Rao (2009)), and attack the problem by using the advances of numerical methods for optimal control. Doing so allows us to clearly see the optimal behavior of a standard economic agent under a wide array of feasible parameterizations of the model, as well as provide a straightforward, viable remedy to the counterfactual behavior of the lifetime labor supply path of impatient agents, as will be detailed below. Thus, we further hope to encourage economists to deeply analyze continuous-time bounded control problems and their applications in economics, partly because of the references mentioned in Footnote 5, and partly because optimal control and the theory of differential equations are long- and well-researched leading branches of mathematics, that can be utilized to reduce the computational challenges so often faced by economists working on large scale, intertemporal decision-making models.

The rest of the paper is organized as follows. Section 2 presents a standard CS/LL model with the usual and augmented preferences. Section 3 uses recent advances in numerical optimal control to quickly and efficiently solve a CS/LL model for a large set of permutations of the model parameters. Implications of the model under different assumptions on preferences and discount rate values, are briefly discussed as well. The last section presents the conclusion to this paper.

2 Model: Basic Setup

Time is continuous and denoted by t. The representative agent enters the workforce at birth ($t = 0$). Let $Q(t)$ denote the probability of surviving until age t, which is a strictly positive and decreasing C^1

[5] There are a number of important reasons where continuous-time optimal control models may be more appropriate to use than their discrete-time analogues (see, e.g., Angeletos et al. (2001, p. 65), Barro and Sala-i-Martin (2004, p. 411), Caliendo (2011, p. 669), and Groth (2013, pp. 343-344)).

[6] We thank Georges Zaccour for bringing this point to our attention.

function.[7] The individual definitely exits the model by age $t = T > 0$. If employed, the agent earns a market-determined constant wage, w, per labor efficiency unit, $\epsilon(t)$.

All wage income not consumed flows into the individual financial asset account $k(t)$, which grows at the market rate of interest, r. The individual starts their life-cycle with no assets, and if he survives till age T, he finishes the life-cycle with no assets either. If an agent dies with positive or negative asset holdings, these assets simply disappear from the model's environment. This is a common assumption when the focus is on the partial equilibrium and micro feature of the model.

We consider a standard intertemporal utility maximization by a representative agent. Preferences over consumption and leisure are given by the instantaneous utility function

$$U(c(t), l(t)) = \frac{(c(t)^{\phi} l(t)^{1-\phi})^{1-\sigma}}{1-\sigma} + z f(t) l(t), \tag{1}$$

where $\sigma > 0$ (and $\sigma \neq 1$) is the inverse elasticity of intertemporal substitution, and $0 < \phi < 1$. The time endowment of agents is normalized to unity, so $l(t)$ is defined as the fraction of time the agent devotes to non-work activity. Function $f(t)$ is assumed to be continuous in time, and non-negative. Here z is a constant which takes the values of either 0, or 1. If $z = 0$, we have standard preferences used in mainstream literature. If $z = 1$, then provided that $f'(t) > 0$, we can say the marginal utility of leisure gets bigger over time relative to the case when z is zero.

Let the rate of time preference be denoted by ρ. The agent's problem can be formulated as

$$\underset{\{c(t), l(t)\}}{Max} \int_0^T Q(t) e^{-\rho t} U(c(t), l(t)) dt \tag{2}$$

subject to the trajectory (or state) equation, control region, and end-point conditions, provided in (3)-(7), respectively, as below:

$$\frac{dk(t)}{dt} = rk(t) + w\epsilon(t)(1 - l(t)) - c(t), \tag{3}$$

$$0 \leq l(t) \leq 1, \tag{4}$$

$$c(t) \geq 0, \tag{5}$$

$$k(0) = 0, \tag{6}$$

$$k(T) = 0. \tag{7}$$

2.1 Examples of the Model Solutions ($z = 0$)

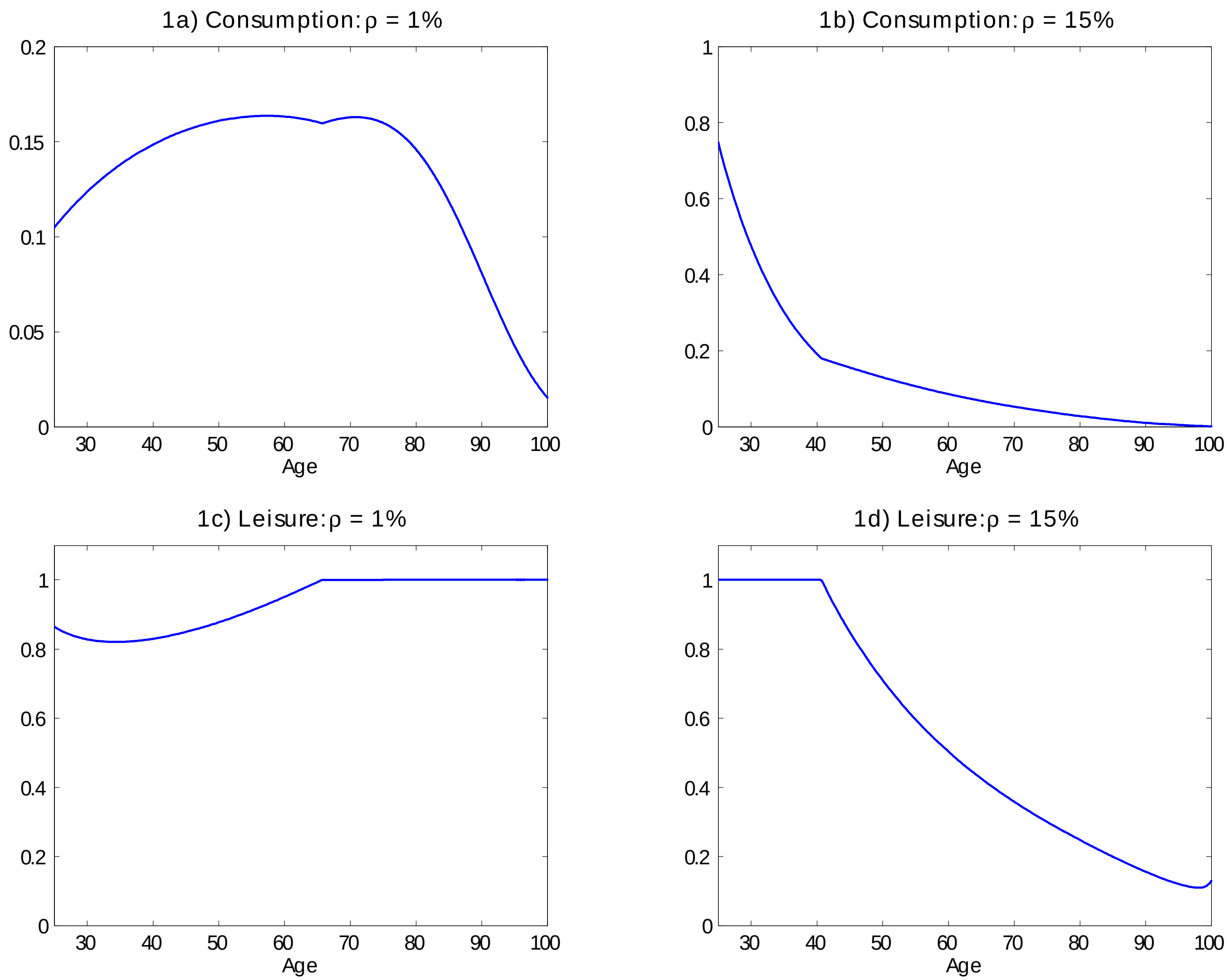
The above problem must be solved via numerical optimal control. Recent advances in numerical optimal control includes the pseudospectral optimal control method (see Ross and Fahroo (2003), Rao (2009)). In this paper, we use a MATLAB toolbox developed by computational scientists Patterson and Rao

[7] Although we will be weighting the utility stream from future consumption by survival probabilities, as done in a number of earlier models, we note that literature has recently considered nonadditive preferences that exhibit a constant absolute risk aversion with respect to life length, as in Bommier (2006).

(2013), which relies on pseudospectral optimal control methods based on Radau collocation points. We augment a standard MATLAB code so it efficiently summarizes the optimal solution structure for an arbitrary set of parameters, thus allowing us to easily spot interesting solutions.

We normalize the wage rate to $w = 1$, and assume the maximum life length is 100 years. As we model agents from age 25 onward, we set $T = 75$. We borrow survival probability $Q(t)$ from Feigenbaum (2008). Efficiency profile, $\epsilon(t)$, is taken from Gahramanov and Tang (2013).

Figure 1: $z = 0$ (standard preferences), $r = 3.5\%$, $\sigma = 3.3$, $\phi = 0.12$.



We set $z = 0$ (standard preferences), $r = 3.5\%$ and $\sigma = 3.3$. The latter two values lie comfortably within a wide range of parameters used in life-cycle, and quantitative studies (Bullard and Feigenbaum (2007); Feigenbaum (2008); Caliendo and Gahramanov (2009)). To generate a realistic labor supply profile, some studies often have to assume a negative discount rate (Bullard and Feigenbaum (2007)), however, to be consistent with most empirical evidence, we strictly consider positive discount rates in this paper. Thus, for a very patient agent ($\rho = 1\%$) we set $\phi = 0.12$, which induces the agent to retire at about age 65.9 (See Fig. 1c). That retirement age is close to the median planned retirement age in the U.S., with the consumption path having a conventional hump shape.

Next, an impatient agent ($\rho = 15\%$), would behave very differently under the above parameter values for r, σ, and ϕ as evidenced from Fig. 1.[8] Fig. 1d shows the impatient agent does not enter the job market for well over fifteen years in a row, preferring to borrow in order to sustain a desired consumption level. After finally entering the job market, the agent keeps working more and more hours over time, with the peak number of hours occurring when he is in his late 90s, working more than 14 hours a day.[9] Thus, even a relatively moderately impatient agent would refuse to start working despite the fact there is no apparent friction in the model to induce him behave this way. The reason behind that behavior is simple: not only does an impatient agent want to consume relatively earlier in life, he also wants to rest relatively earlier in life.

Remark 1. As σ and ϕ are very hard-to-observe parameters, it is possible that high discount rate agents may have different σ and ϕ than low discount rate agents. However, even if a researcher abruptly changes σ and ϕ parameters for the impatient agent in order to generate better results, the extreme intensity of labor supply at old age does not disappear for a sufficiently impatient agent.

Remark 1 is important, as it shows that simply changing the model's free, unobservable parameters is not helpful in generating a more or less realistic labor supply profile if the agent is sufficiently impatient. This has an important implication for the quantitative-theoretical macro studies, where a common practice is to vary unobservable preference parameters to replicate desired empirical data and trends. If a non-trivial fraction of people in reality are indeed highly impatient, then this would create serious challenges to standard calibration exercises.

Remark 2. High discount rates lead to optimal consumption paths that have a large negative slope (see Fig. 1b). This is intuitive, yet empirically, aggregate consumption profiles are not monotonic, but hump-shaped (Feigenbaum 2008).

The implication of Remark 2 is also important because if a large fraction of the population has a high discount rate, a traditional life-cycle model may struggle to generate a realistic aggregate consumption profile. In addition, many agents in reality reveal an abrupt, almost discrete drop at old age consumption, yet nothing of this sort is even remotely apparent from Figs. 1a and 1b. In the next section, we consider alternative preferences to shed more light on a viable remedy to the above challenges.

2.2 Alternative Preferences: $z = 1$

Suppose the correct specification of preferences in (1) is when $z = 1$. Our objective is to find a function $f(t)$, so that: (i) the integrand in the objective functional is a continuously differentiable function in t

[8] To stack the cards against our arguments, we choose a discount rate close to a lower end of the estimates mentioned in the introduction.

[9] Assume there are 16 hours per day available in non-sleep time. An agent who spends about 14.4 hours per day on the job spends about 90% of his non-sleep time at work, as can be seen from a very old agent's leisure consumption depicted in Fig. 1d.

(Kamien and Schwartz 1981, pp. 111-112), and (ii) a more realistic leisure and consumption path are achieved.

We want to stress that according to our model, a researcher may use *any* such functional form for $f(t)$ he wants. For the sake of illustration, let us consider a simple, yet convenient form:

$$f(t) = z_1 + z_2 z_3^t, \tag{8}$$

where z_1, z_2, z_3 are constants. The advantage of the above functional form is that it is continuous, strictly increasing in time, and that by controlling its parameters we can generate a sufficient rise in the marginal utility of leisure at older age, while keeping $f(t)$ (and thus additions to the marginal utility of leisure) for early age nil.

Assume again that the discount rate is high ($\rho = 15\%$), and set $z_1 = 0.000099786002998549\,69$, $z_2 = 2.1399700145031917 * 10^{-7}$, and $z_3 = 1.6237918317754834$. Consequently, the value of $f(t)$ till about age 45 is practically nil, and starts slowly rising till about age 50, yet abruptly spiking thereafter. If we recall from Remark 1, that when $z = 0$ (no $f(t)$ effect), it was not possible to alter the traditional preference parameters (σ and ϕ) to get reasonably realistic consumption paths. Yet with $f(t)$ at play, we can find reasonable values of σ and ϕ to achieve better results. Indeed, let us set $\sigma = 8$ and $\phi = 0.4$. The consumption and leisure profiles are shown in Figs. 2a and 2b, respectively.

Figure 2: $z = 1$ (augmented preferences), $r = 3.5\%$, $\sigma = 8$, $\phi = 0.4$.

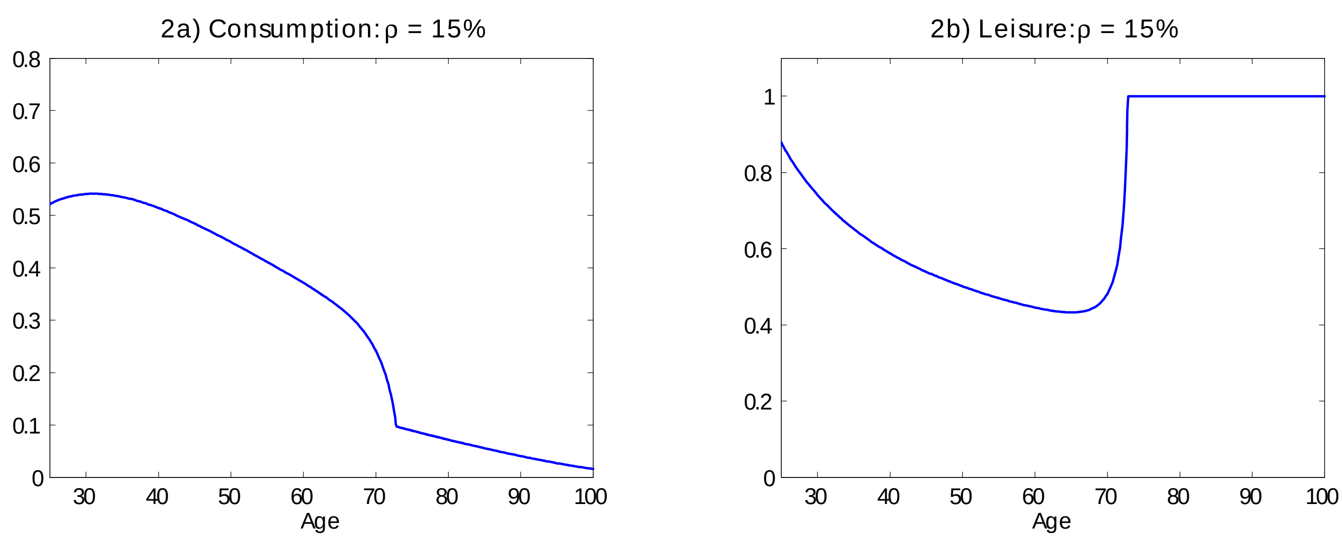


Fig. 2b reveals the impatient agent retires not too late in life, at around age 70. We can also see that unlike in Fig. 1b, the consumption path in Fig. 2a displays a modest hump, occurring quite early in life (about age 31). In addition, at retirement, there is a smooth, yet still very abrupt drop in consumption by the agent. As Huang and Caliendo (2011) point out, observed discrete drops in aggregate consumption data at retirement is mainly due to a subset of the population with very little savings. It is not uncommon to believe that such drops in consumption are likely due to the prevalence of irrational, almost hand-to-mouth Keynesian consumers, yet Huang and Caliendo argue that such discrete drops in aggregate consumption at retirement are anticipated. Our results also indicate that even a perfectly rational, neoclassical agent, with a high degree of impatience and augmented preferences ($z = 1$), will rationally anticipate a "near discrete" consumption drop at old age. Hence, the fraction of irrational, Keynesian consumers in the economy may be overstated.

2.2.1 Further Implications

Suppose the correct specification of preferences in (1) is when $z = 1$, yet a researcher assumes $z = 0$. Further suppose the actual discount rate (ρ) is high, yet the researcher assumes it too low to generate a desired capital-output ratio. It could then be argued that by adjusting other preference parameters (ϕ and σ), the researcher can successfully replicate the observed aggregate consumption and labor supply data. However, it should be noted that a welfare analysis would be skewed. Often policy-makers and researchers encourage various tax/transfer programs, and/or advocate that people retire significantly later than their predecessors (and this advocacy gets even louder as populations age). But if we fail to recognize that some people may be strongly averse to the mere idea of working more when they are old, and/or to the idea of forced saving (because of high discount rate), any policy recommendations based on such welfare analysis may become inefficient.

3 Conclusions

It has been argued the assumption of impatient agents with consistent preferences, presents a serious challenge to a standard, high-frequency intertemporal model of consumption/saving and labor supply. We can easily remedy these challenges by augmenting a standard utility function based on the recent evidence from leisure sciences, at the same time as maintaining a rational choice approach to neoclassical economics. Our findings present a new direction for choice data studies to focus not just on people's preferences over monetary payoffs and conventional consumption, but also on leisure consumption at different ages. Being able to categorize the population by a degree of impatience and time-dependent leisure preferences, will better guide associated welfare analysis and the policy implications of various economic reforms.

References

Angeletos, G.-M., Laibson, D., Repetto, A., Tobacman, J., Weinberg, S. 2001. The Hyperbolic Consumption Model: Calibration, Simulation, and Empirical Evaluation. *Journal of Economic Perspectives* 15(3), 47-68.

Barro, R., Sala-i-Martin, X. 2004. *Economic Growth*, The MIT Press, Cambridge MA.

Bommier, A. 2006. Uncertain Lifetime and Intertemporal Choice: Risk Aversion as a Rationale for Time Discounting. *International Economic Review* 47(4), 1223-1246.

Bommier, A., Leroux, M-L., Lozachmeur, J-M. 2011. Differential Mortality and Social Security. *Canadian Journal of Economics* 44(1), 273-289.

Bullard, J., Feigenbaum, J. 2007. A Leisurely Reading of the Life-cycle Consumption Data. *Journal of Monetary Economics* 54(8), 2305-2320.

Bütler, M., 2001. Neoclassical Life-cycle Consumption: A Textbook Example. *Economic Theory* 17(1), 209-221.

Caliendo, F. 2011. Time-inconsistent Preferences and Social Security: Revisited in Continuous Time. *Journal of Economic Dynamics and Control* 35(5), Pages 668-675.

Caliendo, F., Gahramanov, E. 2009. Hunting the Unobservables for Optimal Social Security: A General Equilibrium Approach. *Public Finance Review* 37(4), 470-502.

Coller, M., Williams, M. 1999. Eliciting Individual Discount Rates. *Experimental Economics* 2(2): 107-127.

Feigenbaum, J. 2008. Can Mortality Risk Explain the Consumption Hump? *Journal of Macroeconomics* 30(3), 844-72.

Feigenbaum, J., Caliendo, F. 2010. Optimal Irrational Behavior in Continuous Time. *Journal of Economic Dynamics and Control* 34(10), 1907-1922.

Feigenbaum, J., Caliendo, F., Gahramanov, E. 2011. Optimal Irrational Behavior. *Journal of Economic Behavior and Organization* 77(3), 285-303.

Feigenbaum, J., Gahramanov, E., Tang, X. 2013. Can High Discount Rates Increase Capital Accumulation? Working Paper, Utah State University.

Feigenbaum, J., Geng, L. 2012. Lifecycle Dynamics of Income Uncertainty and Consumption. *B.E. Journal of Macroeconomics* 12(1), 1-49.

Findley, T., Caliendo, F. 2013. Time Inconsistency and Retirement Choice. SSRN working paper series.

Gahramanov, E. and Tang, X. (2013). Solving for the Retirement Age in a Continuous-time Model with Endogenous Labor Supply. Working Paper, Deakin University.

Gahramanov, E. and Tang, X. (2014). Career Interruptions: A Neglected Aspect of a Neoclassical Model. Working Paper, Deakin University.

Gourinchas, P., Parker, J. 2002. Consumption Over the Life Cycle. *Econometrica* 70(1), 47-89.

Gregory, J., Lin, C. 1992. *Constrained Optimization in the Calculus of Variations and Optimal Control Theory.* Chapman & Hall, London.

Groth, C. 2013. Lecture Notes in Macroeconomics. Mimeo, University of Copenhagen.

Harrison, G., Lau, M., Williams, M. 2002. Estimating Individual Discount Rates in Denmark: A Field Experiment. *American Economic Review* 92(5), 1606-1617.

Heckman, J. 1974. Life Cycle Consumption and Labor Supply: An Explanation of the Relationship between Income and Consumption Over the Life Cycle. *American Economic Review* 64(1), 188-194.

Huang, K., Caliendo, F. 2011. Rationalizing Multiple Consumption-Saving Puzzles in a Unified Framework. *Frontiers of Economics in China* 6(3): 359-388.

Kamien, M., Schwartz, N. 1981. *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management.* Elsevier North Holland, New York.

Kopecky, K. 2011. The Trend in Retirement. *International Economic Review* 52(2): 287-316.

Kydland, F. 1995. Business Cycles and Aggregate Labor Market Fluctuations. In: T. F. Cooley (ed.), *Frontiers of Business Cycle Research*, Princeton University Press.

Ladrón-de-Guevara, A., Ortigueira, S., Santos, M., 1999. A Two-Sector Model of Endogenous Growth with Leisure. *Review of Economic Studies* 66(3), 609-631.

Liechty, T., Genoe, R. 2013. Older Men's Perceptions of Leisure and Aging. *Leisure Sciences: An Interdisciplinary Journal* 35(5), 438-454.

Loewenstein, G., Thaler, R. 1989. Anomalies: Intertemporal Choice. *Journal of Economic Perspectives* 3(4), 181-193.

Morrison, W., Oxoby, R. 2013. The Endowment Effect and Intertemporal Choice: a Laboratory Investigation. *Canadian Journal of Economics* 46(2), 689-704.

Rao, A. 2009. A Survey of Numerical Methods for Optimal Control. In *AAS/AIAA Astrodynamics Specialist Conference*, (Pittsburgh, PA), AAS Paper 09-334.

Ross, I., Fahroo, F. 2003. A Unified Computational Framework for Real-Time Optimal Control *Proceedings of the 42nd IEEE Conference on Decision and Control* 3, 2210-2215.

Warner, J., Pleeter, S. 2001. The Personal Discount Rate: Evidence from Military Downsizing Programs. *American Economic Review* 91(1), 33-53.

Xia, B. 2011. A Simple Explanation of Some Key Time Preference Anomalies. *Canadian Journal of Economics* 44(2), 695-708.