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Competitive Search with Ex-post Opportunism

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Abstract

We consider a frictional market where buyers are uncoordinated and sellers can not commit to a per-unit price and quantity of a divisible good *ex-ante*. By doing so sellers can exploit their local monopoly power by adjusting prices or quantities depending on the case once the local demand is realized. We find that when sellers can adjust quantities *ex-post*, then there exists a unique symmetric equilibrium where the increase in the buyer-seller ratio leads to higher quantities and prices in equilibrium. When sellers post *ex-ante* quantities and adjust prices *ex-post*, a symmetric equilibrium does not exist.

JEL Classification: D40, L10
Keywords: Competitive Search, Price Posting, Quantity Posting

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1 Introduction

Canonical competitive search models (Moen (1997), Acemoglu and Shimer (1999), Mortensen and Wright (2002), Hawkins (2013)) typically assume that sellers are able to fully commit to the posted terms of trade. The only possibility to exploit ex-post opportunities, after the realization of buyers' selections, is to initially post an auction. Kim and Kircher (2015) analyze the implications of first-price auctions and second-price auctions (equivalently, ex-post bidding) and show that the choice of the trading mechanism is crucial for the existence of equilibrium.

The focus of this paper is to characterize the equilibrium, if sellers cannot fully commit ex-ante to the terms of trade. In contrast to Kim and Kircher (2015), we focus on quantity and price posting.

2 Model

Building on the competitive search framework of Moen (1997), we consider a continuum of uncoordinated buyers and sellers, with measures Θ and 1, respectively. Buyers have preferences $u(q)$ over goods produced by sellers who incur a cost $c(q)$, where $u(\cdot)$ and $c(\cdot)$ satisfy usual assumptions of strict concavity and convexity and $c'''(\cdot) > 0$. Any positive measure of sellers posting the same terms of trade, $\omega = (p, q, \theta) \in \mathbb{R}^3_+$, form a submarket where p denotes the per-unit price and θ is the corresponding buyer-seller ratio. We define seller' surplus as $S(p, q) = pq - c(q)$ and buyer' surplus as $B(p, q) = u(q) - pq$. Buyers have access to a continuum of these submarkets and choose which market to participate and all actions are observable to everyone. Within each submarket, buyers and sellers meet according to a matching technology that is homogeneous of degree one, where the seller's (buyer's) meeting rate is $\alpha(\theta)$, $(\alpha(\theta)/\theta)$ with $\alpha'(\cdot) > 0$, $\alpha''(\cdot) < 0$, $\alpha(0) = 0$, $\lim_{\theta\to\infty} \alpha(\theta) = 1$, and $\lim_{\theta\to 0} \alpha'(\theta) = 1$.[1]

Given this structure, we construct a competitive search equilibrium by looking for an optimal deviation of a submarket where all sellers post ω and the rest of sellers in other submarkets post ω^c. We first analyze a situation where sellers post ex-ante

[1] These properties are standard in the matching frameworks of Diamond-Mortensen-Pissarides.

per-unit prices and quantities. We then explore the situation where sellers do not have as much ex-ante commitment. In particular, we consider a situation where sellers post ex-ante per-unit prices and determine the quantity ex-post. Finally, we characterize the equilibrium when sellers post ex-ante quantities and set the per-unit price ex-post.

2.1 Ex-ante Price and Quantity Posting

In this environment, sellers solve

$$\max_{(p,q,\theta)} \alpha(\theta) S(p,q) \text{ s.t. } \frac{\alpha(\theta)}{\theta} B(p,q) \geq \bar{U}, \tag{1}$$

where $\bar{U} > 0$ is the buyer's expected market utility from visiting another seller. It is easy to show that optimality implies an efficient equilibrium quantity $q^* = q^e$ where $u'(q^e) = c'(q^e)$ and an implied per-unit price of

$$p^* = \frac{[1 - \varepsilon(\theta)] u(q^*) + \varepsilon(\theta) c(q^*)}{q^*},$$

where $\varepsilon(\theta) = \theta\alpha'(\theta)/\alpha(\theta)$ is the elasticity of the seller's matching rate.

In a symmetric equilibrium $\omega = \omega^c$ and $\theta = \Theta$ which implies

$$p^* = [1 - \varepsilon(\Theta)] \frac{u(q^*)}{q^*} + \varepsilon(\Theta) \frac{c(q^*)}{q^*}. \tag{2}$$

The buyer's and the seller's expected payoff are $B(p^*, q^*)\alpha(\Theta)/\Theta$ and $S(p^*, q^*)\alpha(\Theta)$, where their surpluses are given by $S(p^*, q^*) = p^*q^* - c(q^*)$ and $B(p^*, q^*) = u(q^*) - p^*q^*$, respectively.

2.2 Ex-ante Price Posting

Prices are posted ex-ante and after buyers decide which submarket to visit, sellers randomly select a buyer to trade with (if more than one buyer) and choose the quantity to produce. Relative to the standard competitive search, now sellers optimally choose q ex-post. Sellers post $\omega = (p, \theta) \in \mathbb{R}^2_+$ ex-ante. Thus to construct the equilibrium, we need to not only account for the optimal deviation in price, but also the deviating sellers' optimal ex-post reaction to their quantity choice given the ex-ante price.

We solve for equilibrium backward, solving for sellers' optimal choice of q given p, and then solving for the competitive search equilibrium choice of p.

Consider the *ex-post* problem where deviating sellers take posted p as given, select a buyer if they meet more than one, and solve the following

$$\max_{q} S(p,q) \text{ s.t. } B(p,q) \geq 0.$$

For interior solutions, the optimal quantity $\tilde{q}$ satisfies

$$p = c'(\tilde{q}) \text{ and } u(\tilde{q}) > p\tilde{q},$$

while in the corner solution, the optimal quantity is given by

$$p = \frac{u(\tilde{q})}{\tilde{q}} \text{ and } c'(\tilde{q}) < p.$$

This yields a one-to-one relationship $\tilde{q}(p)$.

Lemma 1 *For any Θ, the optimal ex-post choice is given by $p = c'(\tilde{q})$.*

Proof. See Appendix. ■

Taking as given the ex-post optimal choice $\tilde{q}$, we solve for the competitive search equilibrium price. To be consistent with previous notations, let $B(p, \tilde{q}(p)) = u(\tilde{q}(p)) - p\tilde{q}(p) \equiv \tilde{B}(p)$ and $S((p, \tilde{q}(p)) = p\tilde{q}(p) - c(\tilde{q}(p)) \equiv \tilde{S}(p)$.

Then deviating sellers solve

$$\max_{p,\theta} \alpha(\theta) \tilde{S}(p) \text{ s.t. } \frac{\alpha(\theta)}{\theta}\tilde{B}(p) \geq \bar{U}.$$

It is easy to show that the optimal solution satisfies

$$\frac{1-\varepsilon(\Theta)}{\varepsilon(\Theta)} = -\frac{\tilde{B}'(p)}{\tilde{S}'(p)}\frac{\tilde{S}(p)}{\tilde{B}(p)}. \tag{3}$$

In a symmetric equilibrium $p = p^c$ and $\theta = \Theta$, which implies $p(\Theta)$. From the above problem, it is a bit more involved to show existence and uniqueness. Fortunately, we can change the problem by substituting for $p = c'(\tilde{q})$ instead and maximize *as if* sellers were choosing $\tilde{q}$ ex-ante. To simplify the notation, let $\tilde{q} = q$ from now on. The problem for sellers then become

$$\max_{q,\theta} \alpha(\theta) S(q) \text{ s.t. } \frac{\alpha(\theta)}{\theta}B(q) \geq \bar{U},$$

where $B(q) = B(c'(q), q)$ and $S(q) = S(c'(q), q)$. The optimal solution $q(\Theta)$ satisfies

$$\frac{1 - \varepsilon(\Theta)}{\varepsilon(\Theta)} = -\frac{B'(q)}{S'(q)} \frac{S(q)}{B(q)}. \tag{4}$$

Proposition 1 *For any given Θ, there exists a unique symmetric equilibrium where all sellers choose q such that $p(\Theta) = c'(q(\Theta))$.*

Posting prices ex-ante with quantities determined ex-post always yields marginal cost pricing, but does not always give efficiency.

Lemma 2 *The equilibrium is generically not efficient. Moreover, an increase in Θ leads to higher q and p in equilibrium.*

These results are in contrast to the standard competitive search equilibrium where sellers post per-unit prices and quantities ex-ante. Efficient quantity q^* is always achieved. With ex-post quantity, efficiency is achieved only if Θ happens to lead to $u'(q(\Theta)) = p(\Theta) = c'(q(\Theta))$. The ability to commit ex-ante to all terms of trade is a very important assumption to obtain efficient outcome. Comparing the allocation of full commitment with ex-ante commitment on price only, note that if Θ^c is such that $q(\Theta^c) = q^*$ and

$$p(\Theta^c)q^* = c'(q^*)q^* = [1 - \varepsilon(\Theta^c)]\, u(q^*) + \varepsilon(\Theta^c)\, c(q^*),$$

the full and partial commitment outcomes are equivalent. But this holds only for a very specific value of Θ^c. Since equilibrium (p, q) are both increasing in Θ, for all $\Theta < \Theta^c$, sellers would prefer to deviate by committing to (p, q) instead of just p, while the reverse is true if $\Theta > \Theta^c$.

We find the partial commitment equilibrium interesting because it has many applications. One perfect example is restaurant meals. Prices are posted (easily found on websites), but the quantity or quality q is implemented ex-post in a bilateral trade between seller and buyer. There are also other markets to which this partial commitment is applicable. The labor market suits this well. Assume that a measure v of vacancies are to be matched with a measure u of unemployed, with $\Theta = v/u$. The surplus from a match is $f(h)$ where h is the hours worked by workers upon a match. Let $c(h)$ be the cost for workers implementing h, and wh be the wage revenue paid from the firm to the worker. Consider setting up the competitive search problem as workers competing by posting (w, θ) to attract firms, and choosing h ex-post. This environment fits perfectly the above setup where $q \equiv h$, $u(q) \equiv f(h)$, $c(h) \equiv c(q)$, and $wh \equiv pq$. All results follow.

2.3 Ex-ante Quantity Posting

Here sellers can post a quantity ex-ante and adjust the per-unit price ex-post after buyers choose which seller to visit. Ex-ante sellers post $\omega = (q, \theta) \in \mathbb{R}_+^2$ to solve

$$\max_{q,\theta} \alpha(\theta) S(p, q) \text{ s.t. } \frac{\alpha(\theta)}{\theta} B(p, q) \geq \bar{U},$$

while ex-post sellers take as given posted quantity and solve

$$\max_{p} S(p, q) \text{ s.t. } B(p, q) \geq 0.$$

Solving the problem backwards we see that sellers are able to extract all the surplus by pricing $p^* = u(q)/q = g(q)$. Note that the seller's pricing decision does not depend on θ.

Sellers take the ex-post pricing rule as given and solve

$$\max_{q,\theta} \alpha(\theta) S(g(q), q) \text{ s.t. } \frac{\alpha(\theta)}{\theta} B(g(q), q) \geq \bar{U}. \tag{5}$$

From the ex-post pricing decision, it is easy to show that

$$q \frac{\partial p^*}{\partial q} = u'(q) - p^*. \tag{6}$$

Hence, $p^* + q\partial p^*/\partial q - c'(q) = 0$ implies an efficient q^* as $u'(q^*) = c'(q^*)$.

Proposition 2 *A symmetric equilibrium does not exist when sellers post quantities ex-ante.*

When sellers post quantities ex-ante and adjust prices ex-post, they choose to post the efficient quantity to attract buyers and are able to extract all the surplus from trade by adjusting prices later. Buyers fully anticipate that sellers' best ex-post choice is to fully extract all of their surplus, which implies that $\bar{U} = 0$, thus buyers do not participate. If sellers were given the choice of what to post ex-ante, no sellers would want to deviate from posting only q ex-ante.

3 Conclusion

We consider a frictional market where buyers are uncoordinated and sellers cannot commit to a per-unit price and quantity of a divisible good *ex-ante*. As in Kim and Kircher (2015) that the choice of the trading mechanism is crucial for the existence of equilibrium. In particular, we find that when sellers post ex-ante prices, there exists a unique symmetric equilibrium where an increase in the buyer-seller ratio leads to higher quantities and prices in equilibrium. When sellers post ex-ante quantities, a symmetric equilibrium does not exist.

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Appendix

Proof of Lemma 1

Define $\bar{q}$ such that $u(\bar{q}) = \bar{q}c'(\bar{q})$ and $\bar{p} = c'(\bar{q})$. We summarize the seller's best ex-post response as follows:

$$p = \begin{cases} c'(\tilde{q}) & \text{for } p \in (0, \bar{p}] \\ u(\tilde{q})/\tilde{q} & \text{for } p \in (\bar{p}, \infty). \end{cases}$$

It is important to highlight that both of these solutions imply a monotone relationship between quantity and price. Notice the following

$$p \leq \bar{p} \Rightarrow \tilde{q}'(p) = 1/c''(\tilde{q}) > 0 \text{ (interior)}, \tag{7}$$

$$p > \bar{p} \Rightarrow \tilde{q}'(p) = \frac{\tilde{q}^2}{\tilde{q}u'(\tilde{q}) - u(\tilde{q})} < 0 \text{ (corner)}. \tag{8}$$

The positive measure of deviating sellers can choose a price that is either in $(0, \bar{p}]$ for an interior solution or in $(\bar{p}, \infty)$ for a corner solution. Define the first possible deviation as $p_1 = c'(\tilde{q}_1)$ and the second possible deviation as $p_2 = u(\tilde{q}_2)/\tilde{q}_2$. For a given value of Θ, the positive measure of deviating sellers can choose an interior or corner price. It is easy to show that the sellers' expected payoff is

$$\pi_1 \equiv \alpha(\Theta)[p_1\tilde{q}_1 - c(\tilde{q}_1)] < \alpha(\Theta)[p_2\tilde{q}_2 - c(\tilde{q}_2)] \equiv \pi_2,$$

while for buyers we have

$$U_1 \equiv \frac{\alpha(\Theta)}{\Theta}[u(\tilde{q}_1) - \tilde{q}_1c'(\tilde{q}_1)] > 0 = \frac{\alpha(\Theta)}{\Theta}[u(\tilde{q}_2) - \tilde{q}_2u(\tilde{q}_2)/\tilde{q}_2] \equiv U_2.$$

It is clear that $\tilde{B}(p) = 0$ at a corner solution, and no buyers would participate in a deviating submarket that offers p_2 and $\tilde{q}_2$, as an ex-post profit maximizing response. Buyers fully anticipate that the best ex-post choice of sellers is to fully extract all of their surplus. It must be that any price as part of a competitive search equilibrium is $p \in (0, \bar{p}]$. The optimal ex-post choice is then $p_1 = c'(\tilde{q}_1) \in (0, \bar{p}]$. In other words, $\tilde{q}_1(p_1)$ is the equilibrium anticipated sellers' response by buyers.

Proof of Proposition 1

An equilibrium has to satisfy equation (4). Let us define the right hand side of this equation as $\chi(q)$ and let $\bar{q}$ such that $B(\bar{q}) = 0$. It is easy to check that $\exists\ \underline{q} > 0$ such that $B'(\underline{q}) = 0$, while $B'(q) < 0$ and $B(q) > 0$ for $q \in (\underline{q}, \bar{q})$. Since $B'(\underline{q}) = 0$, $\lim_{q \to \underline{q}} \chi(q) = 0 < [1 - \varepsilon(\Theta)]/\varepsilon(\Theta)$, and $\lim_{q \to \bar{q}} \chi(q) = \infty$. Therefore, $\exists\ q \in (\underline{q}, \bar{q})$

such that $\chi(q) = [1 - \varepsilon(\Theta)]/\varepsilon(\Theta)$, and equilibrium exists. The equilibrium price $p(\Theta)$ is determined by $p(\Theta) = c'(q(\Theta))$.

To show uniqueness, let $T(q) = B(q) + S(q)$ be the total surplus. Clearly, $T'(q) = B'(q) + S'(q) = u'(q) - c'(q)$ and the equilibrium satisfies

$$\chi(q) = -\frac{[T'(q) - S'(q)] / [T(q) - S(q)]}{S'(q)/S(q)}.$$

$\forall\, q \in [\underline{q}, q^*)$, we have $B'(q) < 0$ since $B'(\underline{q}) = 0$, and so $0 < T'(q) < S'(q)$ and $\chi(q) > 0$ over this interval. Since $T'(q^*) = 0$, $\chi(q^*) > 0$. For all $q \in (q^*, \bar{q}]$, $T'(q) < 0$ and $\chi(q) > 0$ over this interval. This proves that $\chi(q) > 0$ over $q \in [\underline{q}, \bar{q}]$. Observe that $\chi\left(\underline{q}\right) = 0$ since $B'(\underline{q}) = 0$ and all other components of $\chi(q)$ are positive. Note that $\lim_{q \to \bar{q}} \chi(q) = \infty$ as $T(\bar{q}) - S(\bar{q}) = 0$ and all other components are non-zero. For any strictly concave $u(\cdot)$ and convex $c(\cdot)$, $B'(q) = u'(q) - c'(q) - qc''(q) = T'(q) - qc''(q)$. Now $T'(q)$ is monotonically decreasing in q, while $qc''(q)$ is monotonically increasing in q if $c'''(q) \geq 0$. Hence, $\exists!\ q \in [0, \bar{q}]$ such that $B'(q) = 0$. Call this $q = \underline{q}$ and hence $\exists!\ \underline{q}$ such that $B'(\underline{q}) = 0$. Since $\chi(q) > 0\ \forall q \in (\underline{q}, \bar{q})$ with $\chi\left(\underline{q}\right) = 0$ and $\chi(\bar{q}) = \infty$, it must be that $\chi'(q) > 0\ \forall q \in [\underline{q}, \bar{q}]$. Since $\chi(q)$ is monotonically increasing in q over $[\underline{q}, \bar{q}]$, there exists a unique equilibrium $q \in [\underline{q}, \bar{q}]$ for any given Θ.

Proof Lemma 2

Given that $\varepsilon'(\Theta) < 0$**,** $[1 - \varepsilon(\Theta)]/\varepsilon(\Theta)$ is increasing in Θ, and $\chi(q)$ is monotonically increasing over $[\underline{q}, \bar{q}]$. Therefore, an increase in Θ leads to higher q and p.